Abstract
The paper addresses important issues regarding how consumers’ purchase decisions in a product category are influenced by market mix variables in other categories. We propose a structural analysis of multi-category purchase decisions, where we simultaneously model and estimate consumers’ purchase incidence, brand choice and quantity choice decisions across multiple product categories. Additionally, we study the role played by umbrella brands in influencing the decision made by consumers.

We propose a structural model where all the three decisions are derived from the consumer’s global utility maximization. Such structural analysis is important from the perspective of (i) a retailer whose objective is to maximize profits over all product categories, and (ii) a manufacturer whose objective is to maximize profits over its entire product line. Our analysis highlights the importance of studying consumers’ purchase decisions in a multi-category context which can not be addressed by studying each category in seclusion.

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A Multi Category Brand Choice, Purchase Incidence and Purchase Quantity Model to Investigate Cross Promotional Effects of Umbrella Brands

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The Strategic Dimensions of Information Systems Capability: An Evolutionary and Resource-based View

1. Introduction

1.1 Why studying cross categories effects on umbrella brands is important?

While shopping for frequently purchase products, consumers tend to buy from multiple categories such as coffee, cereal, detergent, paper towels; etc. A given shopping trip involves three interrelated decisions by a consumer – first which product categories to purchase (purchase incidence decision or PI), within purchased categories, which brand to choose (brand choice decision or BC) and finally how much quantities to buy for the chosen brands in each category (purchase quantity decision or PQ). Given simultaneous purchases in multiple categories, marketing mix variables in one category may have effect on purchase behavior in other categories. For example, a price reduction on cake frosting might induce a consumer to make a purchase in cake mix and vice versa. However, it is not necessary that demand for products across categories is correlated. For example, demand for Brownies may be independent of demand for Mashed Potatoes.

There are manufacturers in many consumer packaged goods categories that have long product lines in multiple categories. For example, General Mills and Kraft – have product lines that span a wide range of product categories-Cake mix, Cake frosting, Breakfast cereal, Frozen food etc. manufactures have product lines not only across multiple categories but some of the brands across these categories share the same name and are called umbrella brands. For example- Betty Crocker and Pillsbury are popular umbrella brand names used by General Mills across a large number of product categories, while Kraft and Oscar Mayer are popular umbrella brand names used by Kraft Foods across many categories. While brands within each product category are well-defined substitutes, the inter-relationship between categories can be diverse. The effect of marketing mix variables in one category, on demand for products in other categories in presence of umbrella brands, are likely to be different than when the categories do not have such umbrella brands.

A multi-product manufacturer with umbrella brand names has a strategic need to understand correlations in market demand for its product offerings across the product categories. Such an understanding will assist in better coordinating its pricing and promotion decisions for its product lines in the related categories.

1.2 Research Objective

We propose a structural model of three decisions-Purchase Incidence, Brand choice and Purchase Quantity, based on consumer’s basket utility maximization. The structural model would enable us to know the impact of market mix variables of one brand on the sales of brands in other categories. Based on our model, we will show how own and cross category effects of marketing mix variables differ across umbrella vs. non umbrella brands across two categories. Model is estimated on pairs of complements (Cake mix and Cake frosting) and substitutes (liquid and...
powder detergents) product categories. We would also show that how these differences depend on whether categories are complements or substitutes.

1.3 Related Literature

There are 4 streams of research in consumer purchase decisions in a multi-category framework. The first stream consist of papers that have only investigated purchase incidence decision (Manchanda et al 1999). The brand level effects are absent in these papers and therefore, umbrella branding effects are ruled out. The second stream consists of papers that have investigated purchase incidence and brand choice decisions (Song & Chintagunta 2007, Yu Ma and Seetharaman 2005, Mehta 2006). Yu Ma and Seetharaman investigated umbrella branding effects, however, their model did not account for coincidence effects in joint purchase incidence and brand choice decisions. Song and Chintagunta and Mehta did not model both the purchase quantity decision and the umbrella branding effects. The third stream consists of research on brand choice decisions across categories (Erdem 1998, Singh, Hansen and Chintagunta 2006, Ainslie and Rossi 1998, Seetharaman, Ainslie and Chintagunta 1999). The cross category effects are captured by allowing correlations among preference parameters in brand choice across categories. The multi-category brand choice models do not model the impact of each brand's market mix on the category purchase incidence decision. The Fourth stream consist of papers that have investigated all three decisions-PI, BC and PQ (Song and Chintagunta 2006), however, Song and Chintagunta imposes a specific functional form of the direct basket utility that results in purchase incidence decision in one category to be independent of market mix variables in other categories. Their model also did not study the umbrella branding effects.

Our contribution is to model and estimate three interrelated decisions structurally and study the umbrella branding effects across multiple categories. Understanding consumers' purchase quantity decision, along with purchase incidence and brand choice, is very important from manufactures perspective because optimal pricing and promotion policy can only be used if quantity sensitivity with respect to marketing mix variables across categories is fully understood.

Study of umbrella branding allows us to estimate cross category effects at the brand levels. It shows how successful is an umbrella branding strategy for manufacturers and what can be done in terms of individual brand level promotion to increase the overall profits.

2 Model Formulation

2.1 Theoretical Specification of the Purchase Incidence, Brand Choice and Purchase Quantity decisions for a 2 category case

This section is an extension of the theoretical part of model discussed in Mehta (2006). In Mehta (2006), only the purchase incidence and the brand choice decisions were discussed. We extend that theoretical framework to include the purchase quantity decision as well. Consider a consumer who can possibly make a purchase in two product categories in a store on a shopping trip. We define product category \( l = 1, 2 \) as a set of brands \( j = 1, 2, 3, \ldots, J_l \). Let the observed covariates on the consumer's shopping trip be: the total basket expenditure in dollars, \( y \); unit price of a brand \( j \), in category \( l, p_{yj} \) and the perceived quality of brand \( j \), in category \( l, \psi_{yj} \). We aggregate rest of the categories in the shopping basket into a composite commodity whose quality is \( \psi_z \) and the price is \( p_z \).
Given the observed covariates on a shopping trip, the consumer's problem is to make three interrelated decisions—PI, BC and PQ. In order to specify the solution to this problem, we define the consumer’s direct basket utility as a function of her perceived qualities, \( \{ \psi_{ij} \} \) and purchased quantities, \( \{ x_{ij} \} \) of all brands. After converting purchase quantity into budget share \( s_{ij} \) (where the budget share of a brand is related to its purchase quantity as \( s_{ij} = p_{ij}x_{ij}/y \)) and the price into quality adjusted price (which is related to its quality and price as \( p_{ij}^* = p_{ij}/\psi_{ij} \)), the consumer's utility maximization problem can be written as

\[
U \left( \sum_{j=1}^{J} s_{ij}^* y, \sum_{j=1}^{J} s_{2j}^* y, s_z^* \right)
\]

subject to:

1. \( \sum_{j=1}^{J} s_{ij}^* + \sum_{j=1}^{J} s_{2j}^* + s_z^* = 1 \) (budget constraint)

2. \( s_{ij}^* \geq 0, s_{2j}^* \geq 0 \) for all \( j \in (1...J), l \in \{1,2\} \) (non-negativity constraints)

where

- \( s_{ij}^* = p_{ij}^* x_{ij}^*/y = p_{ij} x_{ij}/y = s_{ij} \),
- \( s_{2j}^* = p_{2j}^* x_{2j}^*/y = p_{2j} x_{2j}/y = s_{2j} \),
- \( s_z^* = p_z^* z^*/y = p_z z/y = s_z \)

Given the above problem we next specify the properties of the solutions. It is important to note that in this framework, the consumers first decide whether to purchase in a category or not, and conditional on the purchase, they decide which brand to choose, and the purchase quantity of the chosen brand.

2.2 Solution to the Utility Maximization

First, conditional on purchase in category \( l \), the consumer’s brand choice decision is independent of the specification of the direct utility \( U \). Second, the brand choice decision in a purchased category is independent of the qualities/prices of brands in other categories. Note that this naturally restricts the umbrella branding effects at the brand choice level since it implies that the brand choice decision in one category has no bearing on the brand choice decision in the other category. However, this is only true at the observational level for a given consumer. We will relax this restriction later in section at the population level by assuming correlated heterogeneity in the quality indices of brands in both categories \( \{ \psi_{ij} \} \) at the population level. Third, the purchase incidence condition and the purchase quantity for any category is a function of the quality adjusted prices of brands in that category (the own effects) and the quality adjusted prices of brands in the other purchased category (the cross effects). Further, they do not depend on the qualities or prices of brands in other non-purchased categories. Fourth, the joint category purchase incidence and purchase quantity conditions only requires the specifications of ‘well behaved’ demand functions, \( \{ \varphi(l, q, z^*, y) \}_{l=1}^{2} \) and corresponding budget share function \( \{ \varphi_{l}^* (q, z^*, y) \}_{l=1}^{2} \) for both categories \( l=1, 2 \), in which (i) there is only one brand per category; (ii)
a purchased category \( l \) is assumed to charge the quality adjusted price of the chosen brand \( k \) in that category, that is, \( q_l = p^*_k \). (iii) a non-purchased category \( l \) is assumed to charge the threshold quality adjusted price \( q_l = R^*_l \). This result is crucial since it obviates the need to derive the joint purchase incidence conditions using the first order conditions that follow from maximization of the direct utility \( U \). (Detailed proofs of the above utility maximization problem are given in Mehta-2006)

2.3 Discussion of Own and Cross effects

In this section, we will discuss the own and cross effects of prices in the purchase decisions. We will discuss these effects for both complementary and substitute categories. We will first discuss the own and cross effects of market mix at the category level, assuming only one brand per category. What we mean by category level is that there is a reduction in price across the whole category and not specific to any brand and we want to see the change in the purchase incidence probability of both categories and the change in the budget share across both categories. The objective of this discussion is to highlight the factors that play a role in the purchase incidence and budget share decisions (hence purchase quantity) of a category when there is a change in the price of the same category and compare that with the case when there is a change in the price of another category. Following that, we will relax the restriction of one brand per category and discuss the own and cross effects in the purchase decisions of categories at the brand level. Here, we will discuss what differentiates a brand from other brands in its ability to generate cross category effects in brand choice, purchase incidence and budget share decisions.

What we mean by that is – if there is a change in the price of a given brand in a given category, then how does it impact the purchase incidence probability of the categories, the brand choices in both categories the budget share of brands in both categories.

We assume a very simple form of budget share function for brands in two categories

\[
s_1 = a_1 + b_{11} \ln q_1 + b_{12} \ln q_2
\]

\[
s_2 = a_2 + b_{21} \ln q_1 + b_{22} \ln q_2
\]

Where

\[b_{11}, b_{22} < 0; b_{12} = b_{21} < 0\]

\[b_{11}b_{22} > b_{12}b_{21}\]

The following figure shows the region corresponding to each possible combination regarding purchase decision in both categories in price space \((\ln q_1, \ln q_2)\). In other words, when both categories are purchased (A11), when only category 1 is purchased (A10), when only category 2 is purchased (A01) and when none of the categories are purchased (A00).
The intuition is that when both the prices are high none of the categories are purchased and when both the prices are low both categories are purchased. For the intermediate range of prices, only one category is purchased depending on the price of that category. The thresholds $R_1$ and $R_2$ are such that none of the categories are purchased. To get $R_1$ and $R_2$, we have to solve the following equations

\[ s_1(q_1 = R_1, q_2 = R_2) = 0, s_2(q_1 = R_1, q_2 = R_2) = 0 \Rightarrow R_1 = \frac{a_1 b_{12} - a_2 b_{21}}{b_{11} b_{22} - b_{12} b_{21}}, R_2 = \frac{a_1 b_{12} - a_2 b_{11}}{b_{11} b_{22} - b_{12} b_{21}} \]

next specify the boundary of the four regions

**Region A00**: None of the categories are purchased

\[ A00 \Rightarrow \ln q_1 \geq R_1 = \frac{a_2 b_{12} - a_1 b_{22}}{b_{11} b_{22} - b_{12} b_{21}} \quad \text{and} \quad \ln q_2 \geq R_2 = \frac{a_1 b_{12} - a_2 b_{11}}{b_{11} b_{22} - b_{12} b_{21}} \]

**Region A10**: Only category 1 is purchased

\[ A10 \Rightarrow \ln q_1 \geq \frac{a_2 b_{12} - a_1 b_{22}}{b_{11} b_{22} - b_{12} b_{21}} \quad \text{and} \quad \ln q_2 \geq \left( \frac{a_1 + b_{12} \ln q_2}{b_{11}} \right) \]

**Region A01**: Only category 2 is purchased

\[ A01 \Rightarrow \ln q_1 \geq \left( \frac{a_1 + b_{12} \ln q_2}{b_{11}} \right) \quad \text{and} \quad \ln q_2 \geq \frac{a_1 b_{12} - a_2 b_{11}}{b_{11} b_{22} - b_{12} b_{21}} \]

**Region A11**: Both categories are purchased

\[ \ln q_2 < \frac{a_1 + b_{12} \ln q_1}{b_{12}} \quad \text{and} \quad s_2 > 0 \Rightarrow \ln q_1 < \frac{a_2 + b_{21} \ln q_1}{b_{22}} \]

### 2.4 Category Level Effects

**a) Purchase incidence**: Consider the case where there is a decrease in price in the category 1. As far as the own effects are concerned, the increase in purchase incidence probability for the category 1 comes from the following 2 sources: those who were not purchasing either category 1 or 2, that is, the area A00; and those who were only purchasing in category 2, that is, the area A01. A decrease in the price in the category 1 increases the utility in the category 1, but keeps the threshold utilities for both these regions unchanged. Thus a decrease in the price in the category 1 would cause the utility of category 1 exceed the threshold utility for the category 1 for some marginal consumers in A00 and they will make a purchase the category 1. Similar argument holds for the consumers in the region A01 who will now move into A11. At a given initial prices, the magnitude of price reduction required to move consumer from A01 to A11 is less than the magnitude of price reduction required to move consumer from A00 to A10. In other words, the threshold utility for the category 1 in the region A01 is smaller than the threshold utility for the category 1 in the region A00. Therefore, there will be a greater influx of consumers from A01 than from A00 if the sizes of these two regions were the same.

If there is price cut in the category 2, the purchase incidence in the category 1 will increase from one source: from those consumers who have purchased only category 2 (A01) and now will purchase both the categories 1 and 2 (move to the region A11). This comes from the cross category effects of the purchase incidence in category 1 in the region A01. If there is a price decrease in the category 2, it will decrease the threshold utility for the category 1 in the region A01, thus increasing the purchase probability of category 1. The decrease in threshold utility for
the category 1 depends on the parameter $b_{12}$. The greater the value of $b_{12}$, the greater will be the decrease in the threshold utility for the category 1 and greater will be the increase in the flux of consumers from A01 to A11.

This shows that for the own effects, the demand comes from 2 sources; and for the cross effects, it comes from one source. Further, in the own effects, the purchase incidence increases by increasing the category utility, but in the cross effects, the purchase incidence increases by lowering the threshold utility. Finally, depending on the value of $b_{12}$, since $b_{12}$ is lower than the diagonal terms, the increase in utility is different than the decrease in threshold utility in cross effects.

Note that the cross effects will be asymmetric. If there is a price decrease in category 1, it decreases the threshold utility for category 2 consumers in A10, which increases the flux of consumers from A10 to A11, which increases the purchase probability of category 2. If there is a price decrease in category 2, it decreases the threshold utility for category 1 in A01, which increases the flux of consumers from A01 to A11, thereby increasing the purchase probability of category 1. Note that the threshold utilities for category 1 in A01 and the threshold utility of category 2 in A10 are different. This implies that the cross effects will be asymmetric.

(b) Budget Share (Purchase Quantity): If there is a price decrease in the category 1, all consumers who were purchasing the category 1 (regions A10 and A11) will face this price decrease. Since the budget share (purchase quantity) of such consumers in the category 1 is inversely related to its price, this will increase their budget share (quantity) of the category 1. However, if there is a price decrease in the category 2, only the consumers who are in region A11 will increase their quantity of the category 1. Note that in the budget share (quantity) expression, there are cross effects from the category 2's utility in the region A11. However, in the region A10, the quantity of the category 1 bears no cross effects from the utility in the category 2. Thus, only a subset of the category 1 consumers will increase their purchase quantity. Another difference is that the parameters affecting the own effects are different than the parameters affecting the cross effects. Cross effects can only increase if there is a large fraction of consumers who purchase both categories and not just one. Cross effects as measured by quantity elasticities across 2 categories will be asymmetric because expressions for budget share for 2 categories are different.

2.5 Brand specific effects: we will relax the restriction of one brand per category to see how these cross effects vary across brands in the categories.

(a) Purchase incidence: What causes difference in the cross effects across brands: Consider the case where there are $J_1$ brands in the category 1 and $J_2$ brands in the category 2. Consider the cross category effect on the purchase incidence of the category 2 if there is a price cut of the brand $k_i$ in the category 1. Price cut in the brand $k_i$ in the category 1 will lead to two things in the region A10: the consumers who were previously purchasing brand $k_i$ will continue to purchase it at the lower price and some consumers in the category 1 will switch to the brand $k_i$ who were purchasing other brand before the price cut in $k_i$. Now some of these consumers who have purchased brand $k_i$ at a lower price in the region A10 would have their threshold utilities for the
category 2 lowered. Now if the utility of the optimal brand in the category 2 in the region A10 exceeds this new threshold utility for category 2 in the area A10, this implies that some of these brand \( k_1 \) consumers in A10 would purchase in category 2 and move to A11. Now the question that arises how much does the movement of consumer from A10 to A11 depend on whether there was a price cut in the brand \( k_1 \) in the category 1 or another brand \( j_1 \) in the category 1. First is simple, the share of brand \( k_1 \) or brand \( j_1 \) in category 1 – the greater the share, the greater will be the number of consumers purchasing these brands whose threshold utility for category 2 will be lowered, thus greater will be the number of consumers who will move from A10 to A11. The second depends on whether the consumers who have a high utility for brand \( k_1 \) also have a high utility for brands in category 2. Note that if there is an inverse correlation, it implies that all the brand \( k_1 \) consumers in area A10 will have low utilities for brands in the category 2, thus making it difficult for such consumer’s utilities to exceed the threshold utility of category 2. Now, if the preference for the brand \( k_1 \) is positively correlated with one or more brands in the category 2, it implies that consumers who have purchased \( k_1 \) in category 1 are those who have a high preference for \( k_1 \) – these consumers will also have a high preference for positively correlated brands in category 2. This implies that the utility for brands in category 2 will be high for such customers, thus making it easy for their utilities to exceed the threshold category 2 utility after a cut in the price of \( k_1 \). Thus, the flux of consumers from A10 to A11 depends on how many consumers in A10 have purchased brand \( k_1 \) and whether \( k_1 \) is positively correlated with the brands in category 2 or not. Thus, if the size of \( k_1 \) in A10 is small or if \( k_1 \) is negatively correlated with brands in category 2, the impact of \( k_1 \)'s price on purchase incidence of category 2 will be limited.

(b) Budget share cross effects: we will discuss the effect on the budget share of the brand \( k_2 \) in the category 2 if there is a price cut for the brand \( k_1 \) in the category 1. The price cut in the brand \( k_1 \) in the category 1 will lead to increase in the budget share (quantity) of brand \( k_2 \) in category 2 only for those consumers in A11 who have purchased both \( k_1 \) and \( k_2 \). Note that for the consumers in A01, who are purchasing \( k_2 \), have no impact on their budget share (quantity) since their budget share (quantity) does not depend on the prices of brands in category 1. Further, Note that the budget share (quantity) of those consumers who have purchased \( k_2 \) but have purchased some other brand \( j_1 \) in A11 will not change since the budget share (quantity) for the brand \( k_2 \) in this region only depends on the price of their purchased brand \( j_1 \) in category 1 and not \( k_1 \). Thus the change in budget share (quantity) of consumers purchasing \( k_2 \) with a price cut in \( k_1 \) at the population level depends on: the number of consumers purchasing \( k_2 \) who have also purchased \( k_1 \). Note that this depends on the positive correlations between \( k_1 \) and \( k_2 \). The greater the correlation, the greater will be the chance that consumers would have purchased both together. Thus, if a multi product manufacturer has successfully employed umbrella branding, it would imply that the consumers who prefer one brand in category would also prefer the same brand in a different category. Thus, we would expect purchase quantity elasticities to be higher for umbrella brands.
(c) **Brand choice cross effects:** it is important to note that if the brand \( k_1 \) decreases its price in the category 1, the consumers who had already purchased in the category 2 will not change their brand choice decision. This is because the model does not allow for cross category effects in brand choice decisions at the individual consumer level. However, since we allow for brand preferences to be correlated at the population level, the brand choices in category 2 will change at the aggregate level if there is a price cut in the brand \( k_1 \) in the category 1. This comes from the fact that if there is a price cut of the brand \( k_1 \), then there will be consumers in A10 who have purchased \( k_1 \) will now purchase a brand in category 2 and move to A11. Thus, the population of consumers who purchase category 2 will consist of those consumers who were already purchasing category 2 before \( k_1 \)’s price drop (and whose brand choice decisions in category 2 have not changed) plus the new influx of consumers from A10 who have purchased \( k_1 \) along with a brand in category 2. This brand in category 2 will be the brand with which \( k_1 \) has a positive correlation in the preferences. Thus, if \( k_2 \) was such brand, it would imply that the fraction of consumers in A11 and A01 purchasing \( k_2 \) will increase – thus implying that the brand choice probability in category 2 for \( k_2 \) will increase. If \( k_1 \) is more positively correlated with \( k_2 \) as compared to other brands in category 2, it would imply that the new influx of consumers will purchase \( k_2 \) more than other brands in category 2 and thus brand choice probability of \( k_2 \) will increase.

3 **Stochastic Specification**

In order to characterize the stochastic specification of the 3 decisions, we would need to specify the following: first, the quality indices \( \psi_{i,j} \) of all brands \( j = 1,2,3,...J \), in both categories \( l = 1,2 \); second, the quality adjusted price of the composite commodity \( p^{*} \); and finally, the budget share (demand) functions for both categories \( \{ s, q_{1}, q_{2}, p^{*}, y \} \).

3.1 **Specification of the Quality Indices**

We specify the consumer’s quality index of brand \( j \) in category \( l \) similar to that used by Mehta (2006), as

\[
\psi_{i,j} = \exp \left( \frac{\alpha_{j} + \beta_{j} F_{j} + \varepsilon_{i,j}}{\mu_{l}} \right)
\]

In the above equation, \( \alpha_{j} \) is the brand dummy, \( F_{j} \) is a vector of explanatory variables that impact the preference for brand \( j \) and the sub-utility of category \( l \). The econometrician’s error \( \varepsilon_{i,j} \) is assumed to extreme value as \( \varepsilon_{i,j} \sim \text{Extreme value}(0, \sigma_{i,j}^{*}) \) that is independent across all brands, categories, consumers and shopping trips. The parameters \( \beta_{j} \) represent the sensitivities of explanatory variables on the preference for the brand, and the parameter \( \mu_{l} \) is the inverse of consumer’s quality sensitivity in category \( l \) (which is restricted to be positive). In order to relax the restriction of the brand choice decisions to be independent at the population level, we assume
the intercepts of all brands across both categories \( \{ \alpha_{t,j} \} \) and the quality sensitivities of both categories \( \{ \mu_t \} \) to be correlated at the population level.

The explanatory variables \( F_{ij} \): These are consisting of brand specific and category specific variables. The vector of the explanatory variables in the quality indices can be represented as

\[
\beta_i F_{i,j} = \beta_{pro} \text{Pro}_j + \beta_{loy} \text{GL}_j + \beta_{inv} \ln(\text{Inv}_l + 1)
\]

where \( \text{Pro}_j \) indicates presence of promotions for brand \( j \) in category \( l \), \( \text{GL}_j \) represents the Guadagni and Little's brand loyalty, \( \text{Inv}_l \) denotes the household's inventory of category \( l \) on the given purchase occasion, and \( \{ \beta_{pro} , \beta_{loy} , \beta_{inv} \} \) are the sensitivity parameters that are assumed to be category specific. Specification of the quality adjusted price of the composite commodity is given as

\[
p^*_z = \exp(-\varepsilon_z \mu_z)
\]

Where \( \varepsilon_z \) is assumed to be an extreme value \( (\varepsilon_z \sim (0,1)) \) that is independent across all consumers and all purchase occasions and also independent of the errors \( \varepsilon_y \). The term \( \mu_z \) is the factor by which the shocks \( \varepsilon_z \) are scaled.

### 3.2 Specification of the budget share functions for the 2 categories

We choose the system of Log Translog demand functions (referred to as LTL henceforth) for the two categories. These demand functions are derived from a LTL indirect utility (using the Roy's Identity) that belongs to the family of the flexible functional forms of the Translog indirect utilities (Pollack and Wales, 1992). The LTL budget share functions for the 2 categories are specified as

\[
s_1(q_1, q_2, p^*_z, y) = \frac{a_1 - b_{11} \ln q_1 - b_{12} \ln q_2 - b_{1z} \ln p^*_z + B_1 \ln y}{1 - B_1 \ln q_1 - B_2 \ln q_2 - B_z \ln p^*_z}
\]

\[
s_2(q_1, q_2, p^*_z, y) = \frac{a_2 - b_{21} \ln q_1 - b_{22} \ln q_2 - b_{2z} \ln p^*_z + B_2 \ln y}{1 - B_1 \ln q_1 - B_2 \ln q_2 - B_z \ln p^*_z}
\]

Where

\[
B_1 = b_{11} + b_{12} + b_{1z} , B_2 = b_{12} + b_{22} + b_{2z} , B_z = -(B_1 + B_2)
\]

Unit prices of the two categories are denoted by \( \{ q_i \}_{i=1}^2 \) and the \( q_3 = p^*_z \) is the quality adjusted price of the composite commodity \( z \) (the 3\(^{rd} \) category). Recall, If category \( l \) is purchased, then \( q_i = p^*_k \), which is the quality adjusted price of the chosen brand \( k \) in category \( l \) and if category \( l \) is a non-purchased, then \( q_i = R^*_l \), which is the threshold quality adjusted price of that category \( l \). In the LTL budget share functions, there are 2 sets of parameter restrictions. The first is that of symmetry of the 2×2 parameter matrix \( B = \{ b_{ij} \} \) that is \( b_{12} = b_{21} \). The second set of restrictions is that the diagonal terms \( b_{11} \) and \( b_{22} \) of the parameter matrix \( B \) should be strictly positive and 2×2 matrix \( B \) should be positive definite.
Substituting \( \ln p^* = (\varepsilon, \mu_z) \) into the two budget share functions, we get the budget share function for the LTL indirect utility

\[
s_1(q_1, q_2, y) = \frac{a_1 - b_{11} \ln q_1 - b_{12} \ln q_2 + b_{12} \mu_z \varepsilon_z + B_1 \ln y}{1 - B_1 \ln q_1 - B_2 \ln q_2 + B_z \mu_z \varepsilon_z}
\]

\[
s_2(q_1, q_2, y) = \frac{a_2 - b_{21} \ln q_1 - b_{22} \ln q_2 + b_{22} \mu_z \varepsilon_z + B_2 \ln y}{1 - B_1 \ln q_1 - B_2 \ln q_2 + B_z \mu_z \varepsilon_z}
\]

The term \( b_{il} \) represents the own effects of category \( l \)'s prices on its purchase incidence decision. The parameter \( b_{il} \) represents the cross effects of category \( i \) on the purchase incidence of category \( l \). If \( b_{il} > 0 \) then purchase incidence probability of category \( l \) increases if the price of category \( i \) decreases (and vice versa if \( b_{il} < 0 \)).

The term \( \mu_z \varepsilon_z \) represents the impact of unobserved shocks on the budget share function of category \( l \). Since the aggregate shocks, \( \varepsilon_z \), are common across the budget share functions of both categories, they result in correlations in the joint purchase incidence conditions, which can be interpreted as the co-incidence effects. The magnitude of the co-incidence effects depends on the absolute magnitude of the parameter \( \mu_z \). Note that if \( \mu_z = 0 \), there are no co-incidence effects and if the absolute magnitude of \( \mu_z \) is large, there will be strong co-incidence effects. The parameter \( B_i \) represents the impact of the total basket expenditure on the purchase incidence decision of category \( l \). Since the purchase incidence probability of a category should increase with the increase in the total basket expenditure, we would expect \( B_i > 0 \).

The parameter \( a_i \) is the intercept in the purchase incidence condition for category \( l \), which represents the attractiveness to purchase in category \( l \). Note that in the estimation, for a given category \( l \), we can only identify \( J_l \) of the following \( J_l + 1 \) intercepts, where the first intercept is \( a_i \) and the \( J_l \) intercepts are the brand intercepts in the quality indices of the brands in category \( l \). Thus, for identification purposes, we set \( a_i = 0 \) for all categories and estimate the \( J_l \) intercepts for all brands in all categories. Next, note that the specification of the budget share requires the specification of both the numerator and the denominator of the budget share function.

### 3.3 Likelihood

We provide a general form of likelihood for Purchase Incidence, Brand Choice and Budget Share function for the case when both categories are purchased. Purchase incidence in category \( l \) is denoted by indicator variable \( I_l = 1 \) or 0 depending on category is purchased or not; Brand Choice is denoted by \( BC \) and budget share denoted by \( S \)

\[
P(I_1 = 1, I_2 = 1, BC_1 = k_1, BC_2 = k_2, S_1 = s_1, S_2 = s_2)
\]

\[
= \left\{ P(BC_1 = k_1, BC_2 = k_2 / I_1 = 1, I_2 = 1) \times \right\}
\]

\[
\left\{ P(S_1 > 0, S_2 > 0, S_1 = s_1, S_2 = s_2 / BC_1 = k_1, BC_2 = k_2) \right\}
\]

\[
\Rightarrow f(s_1, s_2) P(BC_1 = k_1, BC_2 = k_2)
\]
Data:
The pair of product categories that we study are cake mix and frosting and liquid and powdered detergent. We use these two pair of categories for three reasons: (1) one pair of category is complementary and other pair is substitute and provides us with an opportunity to investigate the substantive effects of cross-category complementarities and substitutability's on brands' prices. Manufacturers in both categories and they employ umbrella branding strategies, i.e., use common brand names. This allows us also to investigate the cross category effects of umbrella brands and compare them to the non umbrella brands across the two categories. The data set has following variables at the individual level- total basket expenditure in dollars, \( y \); unit price of a brand \( j \), in category \( l \), \( p_{lj} \) and the perceived quality of brand \( j \), in category \( l \), \( \psi_{lj} \).

4 Model Parameters and Estimation Methodology for a 2 category case:

In this section we will summarize all the parameters in the model.

4.1 Model Parameters

Till now, we have introduced the following parameters. In order to control for unobserved heterogeneity in the parameters, we assume mixture normal heterogeneity. The mixture normal heterogeneity assumes that a parameter is linear combination of two normally distributed random variables.

1. intercepts for all brands \( j=1..J_l \) in both categories \( l=1..2 \) in the quality indices, \( \{c_{lj}, \psi_{lj}\}_{l=1}^{2} \)
2. promotional sensitivities in quality indices for both categories, \( \{\beta_{l,pro}\}_{l=1}^{2} \)
3. state dependence sensitivity in quality indices of both categories, \( \{\beta_{l,LOY}\}_{l=1}^{2} \)
4. impact of each category's inventory on its preference, \( \{\beta_{l,inv}\}_{l=1}^{2} \)
5. inverse of quality sensitivities in brand choice decisions across both categories \( \{\mu_{l}\}_{l=1}^{2} \) - which are restricted to be positive
6. the standard deviations of the normally distributed errors in the quality indices of all brands \( j=1..J_l \) in both categories \( \{\sigma_{lj}\}_{l=1}^{2} \)
7. intercepts in budget share functions of both categories, \( \{a_l\}_{l=1}^{2} \)
8. scale parameter \( \mu_r \) for the aggregate shocks \( \varepsilon_r \) in the budget share function of both categories
9. Three parameters in symmetric positive definite matrix \( B = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \) in budget share functions for both categories. We generate this matrix as a product of two Cholesky matrices as \( B = C_b C_b^T \) where the lower triangular Cholesky \( C_b \) is given as \( C_b = \begin{bmatrix} c_{b11} & 0 \\ c_{b12} & c_{b22} \end{bmatrix} \) and its transpose is given as \( C_b^T = \begin{bmatrix} c_{b11} & c_{b12} \\ 0 & c_{b22} \end{bmatrix} \)}
10. The 2 parameters in the LTL model above and beyond the HTL model are $B_1 = b_{11} + b_{12} + b_{1z}, B_2 = b_{12} + b_{22} + b_{2z}$. In the HTL model, we restrict $B_1 = b_{11} + b_{12} + b_{1z} = 0, B_2 = b_{12} + b_{22} + b_{2z} = 0$.

5 Comparison with Competing Specifications (this section is still in progress)

For the same data set, we compare the predictive power, goodness-of-fit and parameter estimates of the proposed specification with those of the (i) HTL specification used by Song and Chintagunta (1999).

To reiterate the differences in the proposed and competing specifications, recall that in the proposed model, the joint purchases of any two categories $i$ and $l$ at the observational level are explained by two factors: (i) cross effects in incidence conditions of the two categories, where category $i$'s market mix influence the purchase incidence condition of category $l$ only if category $i$ is purchased; (ii) correlation in the incidence conditions of the two categories that stems from co-incidence. As compared to the proposed model, the additive model assumes no cross effects in incidence conditions, that is $b_{il} = 0 \forall i, \forall l \in \{1, L\}$. On the other hand, Manchanda, Ansari and Gupta 1999 over-emphasizes the role of cross effects in the joint purchase incidence conditions by assuming the incidence condition of category $l$ to be a function of cross effects from category $i$, irrespective of whether category $i$ is purchased or not.

5.1 Category level Own and cross effects in purchase incidence and purchase quantity of categories:

The cross effects of categories' market mix variables in purchase incidence decisions are captured by the non-diagonal parameters in the matrix $B = \{b_{il}\}$. A positive (negative) value indicates purchase complementarity (substitutability) between two categories $i$ and $l$. From the parameter estimates, we get positive value of the cross effect parameter in the purchase incidence decisions between cake mix and frosting. The positive sign of cross-effect parameter suggest that both categories share purchase complementarity in purchase incidence decisions.

Next, we discuss the own and cross effects in purchase incidence decisions at the category level of categories in terms of their own and cross incidence elasticities across the consumer population. We get the following pattern of results:

1. For both categories, the cross category incidence elasticities are smaller than the own category incidence elasticities.

2. Cross category incidence elasticities are asymmetric across both categories.

**Purchase Incidence elasticities**

<table>
<thead>
<tr>
<th>Price of</th>
<th>Purchased incidence</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cake mix</td>
<td>Cake frosting</td>
</tr>
<tr>
<td>Cake mix</td>
<td>-0.2101</td>
<td>-0.0176</td>
</tr>
<tr>
<td>Betty Crocker</td>
<td>-0.5168</td>
<td>-0.0451</td>
</tr>
<tr>
<td>Pillsbury</td>
<td>-0.5455</td>
<td>-0.0430</td>
</tr>
</tbody>
</table>
5.2 (a) Brand Level Correlations in the Intercepts:
The correlations across category 1 and 2. We plan to show that whether consumers who have a higher preference for brand in one category have the same for the second category. If categories are complements, a high correlation implies higher purchase complementarity. Those that have a high correlation implies that – their cross effects will be good and there is a flow of brand equity from one to the other. The results shown in the following table confirms this hypothesis. For example, higher intrinsic preference (mean intercept) for the brand 1 in cake mix category implies higher intrinsic preference for the brand 1 in cake frosting category.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>estimate</th>
<th>std error</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean intercept of brand 1 in cake mix in segment 1</td>
<td>-4.7133</td>
<td>0.3022</td>
</tr>
<tr>
<td>mean intercept of brand 2 in cake mix in segment 1</td>
<td>-1.6625</td>
<td>0.21447</td>
</tr>
<tr>
<td>mean intercept of brand 3 in cake mix in segment 1</td>
<td>-2.2529</td>
<td>0.21022</td>
</tr>
<tr>
<td>mean intercept of brand 4 in cake mix in segment 1</td>
<td>-2.5574</td>
<td>0.22515</td>
</tr>
<tr>
<td>mean intercept of brand 1 in cake frosting in segment 1</td>
<td>-8.0538</td>
<td>1.3751</td>
</tr>
<tr>
<td>mean intercept of brand 2 in cake frosting in segment 1</td>
<td>-2.1995</td>
<td>0.57421</td>
</tr>
<tr>
<td>mean intercept of brand 3 in cake frosting in segment 1</td>
<td>-2.7516</td>
<td>0.55312</td>
</tr>
<tr>
<td>mean intercept of brand 4 in cake frosting in segment 1</td>
<td>-4.8953</td>
<td>0.51569</td>
</tr>
</tbody>
</table>

b) Conditional Brand Choice Elasticities: These are given in following table

<table>
<thead>
<tr>
<th>Price of Cake Mix</th>
<th>Cake Mix Others</th>
<th>Betty Crocker</th>
<th>Pillsbury</th>
<th>Duncan Hines</th>
<th>Cake Frosting Others</th>
<th>Betty Crocker</th>
<th>Pillsbury</th>
<th>Duncan Hines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cake Mix others</td>
<td>-2.112</td>
<td>0.234</td>
<td>0.249</td>
<td>0.282</td>
<td>-0.024</td>
<td>0.004</td>
<td>-0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>Betty Crocker</td>
<td>0.5856</td>
<td>-1.410</td>
<td>0.611</td>
<td>0.627</td>
<td>0.019</td>
<td>-0.030</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Pillsbury</td>
<td>0.6516</td>
<td>0.653</td>
<td>-1.573</td>
<td>0.637</td>
<td>0.022</td>
<td>0.0053</td>
<td>-0.028</td>
<td>-0.028</td>
</tr>
<tr>
<td>Duncan Hines</td>
<td>0.783</td>
<td>0.690</td>
<td>0.677</td>
<td>-1.62</td>
<td>0.021</td>
<td>0.011</td>
<td>-0.001</td>
<td>-0.021</td>
</tr>
<tr>
<td>Cake Frosting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>others</td>
<td>-0.041</td>
<td>0.005</td>
<td>0.006</td>
<td>0.003</td>
<td>-2.37</td>
<td>0.2105</td>
<td>0.227</td>
<td>0.227</td>
</tr>
<tr>
<td>Betty Crocker</td>
<td>-0.004</td>
<td>-0.019</td>
<td>0.006</td>
<td>-0.003</td>
<td>1.346</td>
<td>-2.31</td>
<td>1.464</td>
<td>1.432</td>
</tr>
<tr>
<td>Pillsbury</td>
<td>-0.005</td>
<td>0.012</td>
<td>-0.010</td>
<td>-0.006</td>
<td>1.256</td>
<td>1.240</td>
<td>-2.692</td>
<td>1.386</td>
</tr>
<tr>
<td>Duncan Hines</td>
<td>0.006</td>
<td>0.004</td>
<td>0.002</td>
<td>-0.009</td>
<td>0.933</td>
<td>0.901</td>
<td>1.017</td>
<td>-3.47</td>
</tr>
</tbody>
</table>
(c) Conditional Purchase Quantity Elasticities:

<table>
<thead>
<tr>
<th></th>
<th>Price of Others</th>
<th>Betty Crocker</th>
<th>Pillsbury</th>
<th>Duncan Hines</th>
<th>Cake Frosting</th>
<th>Others</th>
<th>Betty Crocker</th>
<th>Pillsbury</th>
<th>Duncan Hines</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cake Mix</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>others</td>
<td>-0.6859</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.082</td>
<td>0.007</td>
<td>-0.01</td>
<td>-0.004</td>
<td></td>
</tr>
<tr>
<td>Betty Crocker</td>
<td>0.00</td>
<td>-0.780</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.015</td>
<td>-0.033</td>
<td>-0.017</td>
<td>-0.012</td>
<td></td>
</tr>
<tr>
<td>Pillsbury</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.804</td>
<td>0.00</td>
<td>-0.011</td>
<td>-0.017</td>
<td>-0.027</td>
<td>-0.022</td>
<td></td>
</tr>
<tr>
<td>Duncan Hines</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.802</td>
<td>-0.091</td>
<td>-0.0218</td>
<td>-0.030</td>
<td>-0.041</td>
<td></td>
</tr>
<tr>
<td><strong>Cake Frosting</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>others</td>
<td>-0.024</td>
<td>-0.003</td>
<td>-0.002</td>
<td>0.003</td>
<td>-1.27</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Betty Crocker</td>
<td>-0.007</td>
<td>-0.015</td>
<td>-0.011</td>
<td>-0.011</td>
<td>0.00</td>
<td>-1.36</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Pillsbury</td>
<td>-0.021</td>
<td>-0.013</td>
<td>-0.021</td>
<td>-0.012</td>
<td>0.00</td>
<td>0.00</td>
<td>-1.2745</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Duncan Hines</td>
<td>-0.007</td>
<td>-0.009</td>
<td>-0.011</td>
<td>-0.018</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-1.332</td>
<td></td>
</tr>
</tbody>
</table>

**Conclusion**

This is the first paper that models and estimates all three interrelated decisions in frequently purchased goods. Our research has implications for manufacturers' optimal pricing and promotion decisions since the parameters are estimated by taking into account of consumers' complete decision process. We are able to break down price and promotion effects at the brand level and can compare how successful umbrella branding strategies are for a given manufacturer. The results have important implications in Anti-trust litigations where correct estimate of elasticities play important part in judgments regarding the effect of firms' pricing and other promotion policies.

**References**


