Preliminary: Comments Welcome

A Test of The Market Efficiency Hypothesis with An Application to Canadian Treasury Bill Yields

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Abstract:

In this paper we propose a new test for efficiency of spot and forward markets where returns are nonstationary and cointegrated. The test for market efficiency is developed within the framework of a vector error correction (VEC) representation of a bivariate vector autoregression (VAR) model. The proposed test includes some of the popular regression-based tests as its special cases. We then apply the test to the Canadian Treasury bill returns. The data used are average yields of three- and six-month Treasury bills at the last tenders of each month from January 1960 to February 1998. Test results indicate that the bill yields are I(1), cointegrated, and consistent with the bill market efficiency hypothesis.

Key Words: Unit roots; Cointegration, Error correction; Term structure of interest rates; GMM estimation

1. Introduction

Treasury bill or bond market is said to be efficient if its expectations about future yields embody available information efficiently. See Fama (1991). Earlier studies on the efficiency of U. S. Treasury bill market include those by Roll (1970), Sargent (1972), and Fama (1976), among others. Their tests employ a regression model of the following form:

$$s_{t+k} = \alpha + \beta f_t + u_{t+k}, \tag{1}$$

where s_{t+k} is the spot interest rate at time $t+k$, f_t the k-period forward rate at time t, and u_{t+k} an error term. If the bill market is efficient and forward rates embody its expectations about future spot rates, then market efficiency under the assumption of a constant risk premium implies that $\beta = 1$ in Equation (1). The constant α corresponds to the negative risk premium. Regression-based tests of the 1970s found that the market efficiency hypothesis was consistent at least with the data on one-month bill yields. The tests are not applicable, however, if the spot and forward rates are not stationary.

Shiller (1979), Sargent (1979), and others took a different strategy in studying the rational expectations hypothesis of the term structure of interest rates. They argue that if the bond market is efficient, the errors made in forecasting the future interest rates must be orthogonal to the information set at time *t*. Thus if forward rates embody optimal forecasts of future spot rates, the bill market efficiency implies a restriction that $? = 0$ in the following regression of forecast errors:

$$s_{t+k} - f_t = a + ?' I_t + u_{t+k}, \tag{2}$$

where I_t is a vector of variables in the information set available at time *t* and *?* a vector of coefficients for the vector variable I_t. The constant a corresponds to the negative risk premium.

Park's (1982) study with Canadian Treasury bill data used the following regression model of three-month changes in three-month spot rates on three-month forward premiums:

$$s_{t+3} - s_t = a + d\,(f_t - s_t) + u_{t+3}. \tag{3}$$

The estimate of d was not significantly different from one and consistent with the bill market efficiency hypothesis. Unknown at the time of the 1982 study, however, is that if the spot rate s_t and the forward rate f_t are I(1), regression tests based on (3) is valid only when spot and forward rates are cointegrated with a unit coefficient, that is, the forward premium, $f_t - s_t$, is I(0).

More recent studies by Campbell and Shiller (1987), Stock and Watson (1988) and Hall, Anderson and Granger (1992), among others, found U. S. bill yields to be cointegrated. If s_t and f_t are cointegrated, Equation (1) is simply the cointegrating relation between the two interest rates and ß a cointegrating coefficient. To test if $ß = 1$ in (1) is then a test for cointegration with a unit coefficient and not necessarily a test for market efficiency. Market efficiency is more than the cointegration between s_t and f_t with a unit cointegrating coefficient; it implies that market predictions of future spot rates incorporate all available information including that which may be contained in past spot and forward rates. On the other hand, if spot and forward rates are cointegrated with a unit coefficient and forward rates embody market predictions of future spot rates, testing if $? = 0$ in (2) or $d = 1$ in (3) is a test for market efficiency.

Campbell and Shiller (1987) modeled long- and short-term interest rates by a vector autoregression (VAR) of change in short rates and the spread in long- and short-term rates and derived the restrictions on the coefficient of the VAR imposed by the expectations hypothesis. For monthly data on U.S. Treasury 20-year bond yields and one-month Treasury bill rates Campbell and Shiller rejected the expectations hypothesis of the term structure.

The primary purpose of this paper is to develop a new test of market efficiency that utilizes the ideas underlying Equations (2) and (3) within the framework of a vector error correction (VEC) representation of a vector autoregressive (VAR) model of nonstationary but cointegrated spot and forward rates. We then apply this strategy to monthly data of three- and six-month Canadian Treasury bills from January 1961 to August 1998 and test if Canadian bill market is efficient.

The plan of the paper is as follows. Section 2 proposes a new test for market efficiency based on a VEC representation of a bivariate VAR model of spot and forward interest rates. In Section 3 we report the results of cointegration tests applied to spot and forward interest rates while in Section 4 we discuss the results of estimating a VEC model by the generalized method of moments (GMM) and testing the market efficiency hypothesis. Section 5 concludes the paper.

2. Testing Market Efficiency with a VEC Model of Spot and Forward Rates

Consider a bivariate VAR of order p:

$$x_t = \Psi z_t + \Phi_1 x_{t-1} + \Phi_2 x_{t-2} + \ldots + \Phi_p x_{t-p} + \varepsilon_t , \qquad (4)$$

where $x_t = (s_t \;\; f_t)'$ is a 2×1 vector of observations on spot and forward interest rates at time t; z_t is a $m\times 1$ vector of observations on exogenous variables including the intercept term; Ψ is a $2\times m$ coefficient matrix; $\Phi_1, \ldots, \Phi_p$ are 2×2 coefficient matrices; and ε_t is a 2×1 vector of normal white noise errors with the mean zero and the covariance matrix Σ. As is well known, the vector x_t can be expressed alternatively in terms of x_{t-1} and $p-1$ lagged differences $\Delta x_{t-1}, \Delta x_{t-2}, \ldots, \Delta x_{t-p+1}$ as

$$x_t = \Psi z_t + \Pi_0 x_{t-1} + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \ldots + \Gamma_{p-1} \Delta x_{t-p+1} + \varepsilon_t , \qquad (5)$$

or

$$\Delta x_t = \Psi z_t + \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \ldots + \Gamma_{p-1} \Delta x_{t-p+1} + \varepsilon_t , \qquad (6)$$

where Δ is the difference operator such that $\Delta x_t = x_t - x_{t-1}$ and

$$\begin{aligned} \Pi_0 &= \Phi_1 + \Phi_2 + \ldots + \Phi_{p-1} + \Phi_p ; \\ \Pi &= \Pi_0 - I ; \\ \Gamma_i &= \textstyle\sum_{j=1}^{i} \Phi_j - \Pi_0 , \quad i = 1, 2, \ldots, p-1. \end{aligned} \qquad (7)$$

When written more explicitly in terms of spot and forward interest rates s_t and f_t, Equation (6) becomes

$$\begin{bmatrix}\Delta s_t\\ \Delta f_t\end{bmatrix} = \begin{bmatrix}\delta_1\\ \delta_2\end{bmatrix} + \begin{bmatrix}\pi_{11} & \pi_{12}\\ \pi_{21} & \pi_{22}\end{bmatrix}\begin{bmatrix}s_{t-1}\\ f_{t-1}\end{bmatrix} + \begin{bmatrix}\Gamma_{11}^{(1)} & \Gamma_{12}^{(1)}\\ \Gamma_{21}^{(1)} & \Gamma_{22}^{(1)}\end{bmatrix}\begin{bmatrix}\Delta s_{t-1}\\ \Delta f_{t-1}\end{bmatrix} + \ldots + \begin{bmatrix}\Gamma_{11}^{(p-1)} & \Gamma_{12}^{(p-1)}\\ \Gamma_{21}^{(p-1)} & \Gamma_{22}^{(p-1)}\end{bmatrix}\begin{bmatrix}\Delta s_{t-p+1}\\ \Delta f_{t-p+1}\end{bmatrix} + \begin{bmatrix}\varepsilon_{t,1}\\ \varepsilon_{t,2}\end{bmatrix}. \quad \textbf{(8)}$$

where it is assumed that the only exogenous variable in the model is that for the intercept term. If s_t and f_t are I(1) and cointegrated, Equation (8) can be written in a VEC representation as

$$\begin{bmatrix}\Delta s_t\\ \Delta f_t\end{bmatrix} = \begin{bmatrix}?_1(s_{t-1}-\alpha-\beta f_{t-1})\\ ?_2(s_{t-1}-\alpha-\beta f_{t-1})\end{bmatrix} + \begin{bmatrix}\Gamma_{11}^{(1)} & \Gamma_{12}^{(1)}\\ \Gamma_{21}^{(1)} & \Gamma_{22}^{(1)}\end{bmatrix}\begin{bmatrix}\Delta s_{t-1}\\ \Delta f_{t-1}\end{bmatrix} + \ldots + \begin{bmatrix}\Gamma_{11}^{(p-1)} & \Gamma_{12}^{(p-1)}\\ \Gamma_{21}^{(p-1)} & \Gamma_{22}^{(p-1)}\end{bmatrix}\begin{bmatrix}\Delta s_{t-p+1}\\ \Delta f_{t-p+1}\end{bmatrix} + \begin{bmatrix}\varepsilon_{t,1}\\ \varepsilon_{t,2}\end{bmatrix}, \quad \textbf{(9)}$$

where s_t and f_t are assumed to have no time trend and the cointegrating equation has an intercept: $s_t = \alpha + \beta f_t$. Clearly, $?_1 = \pi_{11}$, $?_2 = \pi_{21}$, and $\beta = -\pi_{11}/\pi_{12} = -\pi_{21}/\pi_{22}$. The two coefficients $?_1$ and $?_2$ are the adjustment coefficients while β is the cointegrating coefficient.

We are interested in investigating whether spot and forward rates are cointegrated with $\beta = 1$. Applying Johansen's (1988) full-information maximum likelihood (FIML) method to (8), we can determine if the cointegrating rank is one and, if so, estimate the cointegrating equation and test if $\beta = 1$.

Equation (5) or (6) is in the VEC form of a VAR(p) model that is commonly used in the literature. Since the data we analyze are monthly observations on three- and six-month spot rates and three-month forward rates, we reparameterize (4) in terms of x_{t-3} and lagged differences in x_t. For any values of $\Phi_1, \Phi_2, \ldots, \Phi_p$, the following polynomials in L are equivalent:

$$\begin{aligned} & I - \Phi_1 L - \Phi_2 L^2 - \ldots - \Phi_p L^p \\ & \quad = (I - \Gamma_0 L^3) - (\Gamma_1 L + \Gamma_2 L^2 + \ldots + \Gamma_{p-1} L^{p-1})(1-L), \end{aligned} \quad \textbf{(10)}$$

where L stands for the lag operator such that $L^n x_t = x_{t-n}$ for any integer n and

$$\begin{aligned} & \Gamma_0 = \Phi_1 + \Phi_2 + \ldots + \Phi_{p-1} + \Phi_p; \\ & \Gamma_i = \Sigma_{j=1}^{i}\Phi_j, \quad i = 1, 2; \\ & \Gamma_i = \Sigma_{j=1}^{i}\Phi_j - \Gamma_0, \quad i = 3, 4, \ldots, p-1; \end{aligned} \quad \textbf{(11)}$$

or, equivalently,

$$
\begin{aligned}
?_i &= G_i + ?_0, \quad i = 1,\ 2; \\
?_i &= G_i, \quad i = 3,\ 4,\ ...,\ p-1.
\end{aligned} \qquad \textbf{(12)}
$$

Thus we can express x_t in terms of x_{t-3} and $?x_{t-1},\ ?x_{t-2},\ ...,\ ?x_{t-p+1}$ as

$$x_t = ?z_t + ?_0 x_{t-3} + ?_1 ?x_{t-1} + ?_2 ?x_{t-2} + ... + ?_{p-1} ?x_{t-p+1} + e_t, \qquad \textbf{(13)}$$

or

$$x_t - x_{t-3} = ?z_t + ?x_{t-3} + ?_1 ?x_{t-1} + ?_2 ?x_{t-2} + ... + ?_{p-1} ?x_{t-p+1} + e_t. \qquad \textbf{(14)}$$

Shifting (14) forward by three periods yields

$$
\begin{aligned}
x_{t+3} - x_t = {} & ?z_{t+3} + ?x_t + ?_1 ?x_{t+2} + ?_2 ?x_{t+1} \\
& + ?_3 ?x_t + ... + ?_{p-1} ?x_{t-p+4} + e_{t+3}.
\end{aligned} \qquad \textbf{(15)}
$$

Since $?x_{t+2}$ and $?x_{t+1}$ are not yet realized at time t, we express them on the right-hand side of (15) in terms of realized values of x_t and error terms. Shifting (6) forward, we obtain

$$?x_{t+1} = ?z_{t+1} + ?x_t + G_1 ?x_t + G_2 ?x_{t-1} + ... + G_{p-1} ?x_{t-p+2} + e_{t+1} \qquad \textbf{(16)}$$

and

$$?x_{t+2} = ?z_{t+2} + ?x_{t+1} + G_1 ?x_{t+1} + G_2 ?x_t + ... + G_{p-1} ?x_{t-p+3} + e_{t+2}. \qquad \textbf{(17)}$$

Shifting (5) forward by one period gives

$$x_{t+1} = ?z_{t+1} + ?_0 x_t + G_1 ?x_t + G_2 ?x_{t-1} + ... + G_{p-1} ?x_{t-p+2} + e_{t+1}. \qquad \textbf{(18)}$$

Finally, substituting (18) and (16) into (17) for x_{t+1} and $?x_{t+1}$ and substituting (17) and (16) into (15) for $?x_{t+1}$ and $?x_{t+2}$, we write (15) as

$$
\begin{aligned}
x_{t+3} - x_t = {} & Bx_t + C_1 ?x_t + C_2 ?x_{t-1} + ... + C_{p-1} ?x_{t-p+2} \\
& + A_1 z_{t+3} + A_2 z_{t+2} + A_3 z_{t+1} + e_{t+3} + D_2 e_{t+2} + D_3 e_{t+1},
\end{aligned} \qquad \textbf{(19)}
$$

where the matrices B, C_1, C_2, ..., C_{p-1}, A_1, A_2, A_3, D_1 and D_2 are functions of ? 's, G's, and ? :

$$
\begin{aligned}
B &= (I + ?_1 + ?_1? + ?_1 G_1 + ?_2)? \\
C_i &= ?_1 ? G_i + ?_1 G_1 G_i + ?_1 G_{i+1} + ?_2 G_i + ?_{i+2}, \quad i = 1, ..., p-3 \\
C_{p-2} &= ?_1 ? G_{p-2} + ?_1 G_1 G_{p-2} + ?_1 G_{p-1} + ?_2 G_{p-2}, \\
C_{p-1} &= ?_1 ? G_{p-1} + ?_1 G_1 G_{p-1} + ?_2 G_{p-1}, \\
A_1 &= ?, \\
A_2 &= ?_1 ?, \\
A_3 &= (?_1 ? + ?_1 G_1 + ?_2)?, \\
D_2 &= ?_1, \\
D_3 &= ?_1 ? + ?_1 G_1 + ?_2.
\end{aligned}
\tag{20}
$$

Note that all terms on the right-hand side of (19) except the random errors are given at time t.

Writing (19) explicitly in terms of spot and forward interest rates, we obtain

$$
\begin{bmatrix} s_{t+3} - s_t \\ f_{t+3} - f_t \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} s_t \\ f_t \end{bmatrix} + \begin{bmatrix} c_{11}^{(1)} & c_{12}^{(1)} \\ c_{21}^{(1)} & c_{22}^{(1)} \end{bmatrix} \begin{bmatrix} ? s_t \\ ? f_t \end{bmatrix} + ... + \begin{bmatrix} c_{11}^{(p-1)} & c_{12}^{(p-1)} \\ c_{21}^{(p-1)} & c_{22}^{(p-1)} \end{bmatrix} \begin{bmatrix} ? s_{t-p+2} \\ ? f_{t-p+2} \end{bmatrix} + \begin{bmatrix} u_{t+3,1} \\ u_{t+3,2} \end{bmatrix}
\tag{21}
$$

where $u_{t+3,1}$ and $u_{t+3,2}$ are clearly MA(2) error terms. The first equation of (21) is

$$
\begin{aligned}
s_{t+3} - s_t = a_1 + b_{11} s_t + b_{12} f_t + c_{11}^{(1)} ? s_t + ... + c_{11}^{(p-1)} ? s_{t-p+2} \\
+ c_{12}^{(1)} ? f_t + ... + c_{12}^{(p-1)} ? f_{t-p+2} + u_{t+3,1}.
\end{aligned}
\tag{22}
$$

If s_t and f_t are I(1) and cointegrated, we can write (22) as

$$
\begin{aligned}
s_{t+3} - s_t = a_1 + ?(f_t - \mu s_t) + c_{11}^{(1)} ? s_t + ... + c_{11}^{(p-1)} ? s_{t-p+2} \\
+ c_{12}^{(1)} ? f_t + ... + c_{12}^{(p-1)} ? f_{t-p+2} + u_{t+3,1},
\end{aligned}
\tag{23}
$$

where $\mu = - b_{11}/b_{12}$ is the cointegrating coefficient and $? = b_{12}$.

If the bill market is efficient, the bill prices should fully reflect information available to market participants. In its simplest form the efficient market hypothesis is a joint hypothesis of rational expectations and a model of equilibrium returns. If the market is efficient and its assessment at time t of future spot rate s_{t+3} incorporates all available information at time t, we can write

$$s_{t+3} - s_t = E_t\,(s_{t+3} - s_t \mid \Omega_t) + u_{t+3}, \tag{24}$$

where $E_t\,(\cdot)$ stands for the expectation conditional on information at time t, Ω_t is the information set at time t and u_{t+3} is the forecast error or that part of s_{t+3} which is unpredictable at time t. The conditional expectation of s_{t+3} given Ω_t, $E_t\,(s_{t+3} \mid \Omega_t)$, is the optimal (minimum MSE) 3-month ahead forecast of the spot rate at time t.

Following Fama (1976) and others, we assume a model of equilibrium returns that admit a constant risk (or term) premium and posit the following relationship between the forward premium and expected change in spot rates:

$$f_t - s_t = E_t\,(s_{t+3} - s_t \mid \Omega_t) + p, \tag{25}$$

where f_t is the 3-month forward rate implicit in the difference between the six- and three-month bill prices and p stands for the constant risk premium. Equation (25) is the equilibrium relation that determines the forward rate. Combining (24) and (25), we write

$$s_{t+3} - s_t = -p + (f_t - s_t) + u_{t+3}. \tag{26}$$

If the risk premium is constant and Equation (26) holds, the bill market is efficient in the sense that the forward rate reflects all relevant information about the future spot interest rates that is contained in the information set Ω_t.

Comparing (26) with (23), we find the efficient market hypothesis to imply the following testable hypotheses in terms of the parameters in (23):

$$\mu = 1; \tag{27}$$

$$? = 1; \tag{28}$$

$$c_{11}^{(1)} = \ldots = c_{11}^{(p-1)} = c_{12}^{(1)} = \ldots = c_{11}^{(p-1)} = 0. \tag{29}$$

The first hypothesis in (27) is implied by that of cointegration with a unit coefficient while all three hypotheses jointly by the efficient market hypothesis. The term a_1 in (23) corresponds to $\&p$, the negative premium, in (26).

It is clear that Park's (1982) regression-based test with Equation (2) is a special case of testing (28) assuming that (27) and (29) are true, that is, if $? \ ' \ 1$ under the assumption that $\mu \ ' \ 1$ and $c_{11}^{(1)} \ ' \ c_{12}^{(1)} \ ' \ \ldots \ ' \ c_{11}^{(p\&1)} \ ' \ c_{12}^{(p\&1)} \ ' \ 0$. On the other hand, Shiller's (1979) test with (3) is to test (29) assuming that (27) and (28) hold, that is, to assume that $\mu \ ' \ ? \ ' \ 1$ and test if $c_{11}^{(1)} \ ' \ c_{12}^{(1)} \ ' \ \ldots \ ' \ c_{11}^{(p\&1)} \ ' \ c_{12}^{(p\&1)} \ ' \ 0$. Our proposed strategy to test for market efficiency is to test (28) first, that is, if $\mu \ ' \ 1$ and, if so, test jointly (29) and (30), that is, if $? \ ' \ 1$ and $c_{11}^{(1)} \ ' \ c_{12}^{(1)} \ ' \ \ldots \ ' \ c_{11}^{(p\&1)} \ ' \ c_{12}^{(p\&1)} \ ' \ 0$.

3. Testing for Cointegration of Spot and Forward Rates

The underlying data used in this study are monthly series of average yields on three- and six-month Treasury bills at the last auction of each month from 1961:1 to 1998:8, a total of 472 observations. We have computed monthly series of continuously compounded yields on three-month and six-month bills, s_t and S_t, by applying logarithmic transformation to the yield series at annual rates. We have also computed a series of continuously compounded three-month forward rates, defined by $f_t \ ' \ 2S_t \ \& \ s_t$.

Figure 1: Three-Month Spot and Forward Rates

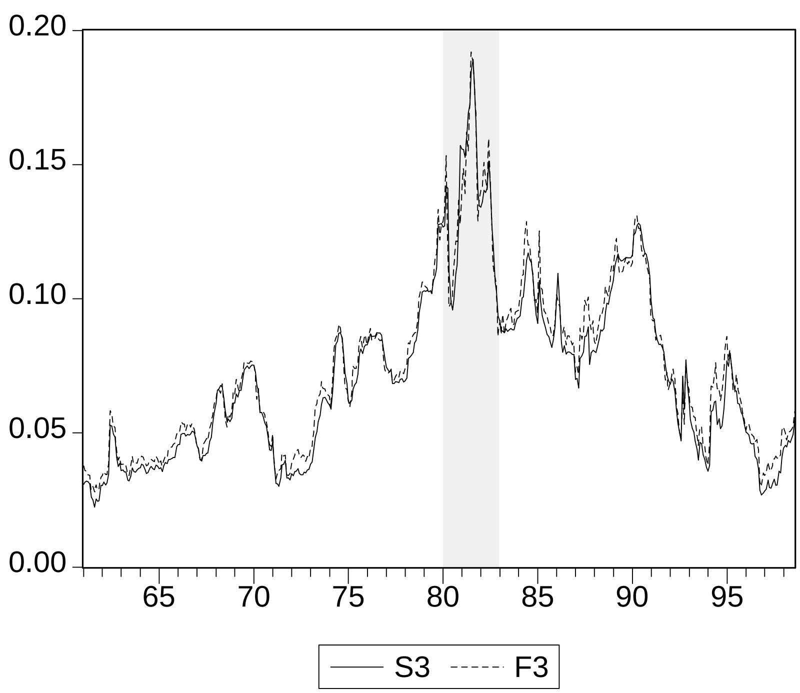

Figure 1 shows three-month spot and forward interest rates in natural logarithm. The two series appear to have moved together, displaying an overall upward trend from the 1960s until the early 1980s and then the familiar cyclicality with a general downward trend with a period of historically high and volatile interest rates from from the late 1970s to the early 1980s. The shaded area in Figure 1 shows this period of interest rate volatility from 1980:1 to 1982:12. In empirical analyses we consider the full sample period from 1961:1 to 1998:8 as well as two subperiods from 1961:1 to 1979:12 and from 1983:1 to 1998:8.

3.1. Unit Root Tests

If spot and forward rates are integrated of order 1, denoted I(1), and there exists a linear combination of the two that is I(0), then the two series are cointegrated of order (1,1) in the sense of Engle and Granger (1987). This section reports on the results of the unit root tests of spot and forward rates.

It is generally accepted in the literature that Treasury bill yields behave like an I(1) process. Suppose that the DGP of an interest rate series is described by

$$r_t = \beta_0 + \rho r_{t-1} + u_t, \tag{30}$$

where r_t stands for the spot or forward rate and u_t is a stationary zero-mean error. The null hypothesis is that r_t is I(1), i.e., $\rho = 1$ (and $\beta_0 = 0$) while the alternative stipulates that r_t is I(0), i.e., $-1 < \rho < 1$ (and $\beta_0 \neq 0$). Following the Dickey-Fuller (1979) strategy, we augment (30) by k lagged changes in the rates to capture the dynamics of u_t and write the testing regression as

$$\Delta r_t = \beta_0 + \phi r_{t-1} + \sum_{i=1}^{k} \beta_i \Delta r_{t-i} + \varepsilon_t, \tag{31}$$

where $\phi = \rho - 1$ and k is large enough to make ε_t serially uncorrelated. The null and alternative hypotheses are $\phi = 0$ and $\phi < 0$, respectively. The augmented Dickey-Fuller (ADF) test statistic is simply the usual "*t*" ratio of the least squares estimate (LSE) of ϕ to its standard error in (31) but the distribution of the statistic is nonstandard under the null.

As the ADF test statistics are sensitive to the lags included, we have computed several of them varying the lag truncation parameter k. Table 1 presents the ADF statistics for three-month spot rate (s), six-month spot rate (S) and three-month forward rate (f) at lags k = 2, 5, 8 and 11. Clearly, the null hypothesis of a unit root is not rejected at the 0.05 level of significance in all cases considered. (Unless stated otherwise, the level of significance for any test reported is 0.05.) Based on the reported ADF statistics and those for the differenced spot and forward rates (computed but not reported here), we conclude that three- and six-month spot and forward rates are all I(1).

Table 1 Augmented Dickey-Fuller Test Statistics for Unit Roots

	Truncation Parameter k			
Variables	2	5	8	11
Sample period: 1961:1 - 1998:8 (T = 452)				
s	-2.562	-2.274	-2.284	-2.236
f	-2.440	-2.158	-2.121	-2.303
S	-2.496	-2.192	-2.180	-2.230
Sample period: 1961:1 - 1979:12 (T = 228)				
s	-0.530	-0.312	-0.046	-0.106
f	-0.420	-0.198	0.059	0.020
S	-0.438	-0.201	0.025	-0.029
Sample period: 1983:1 - 1998:8 (T = 188)				
s	-1.634	-1.368	-1.525	-1.670
f	-1.543	-1.204	-1.309	-1.342
S	-1.541	-1.241	-1404	-1.481

Note: Asymptotic critical value at $\alpha = 0.05$ is -2.868. Davidson and MacKinnon (1993), p. 708.

3.2. Cointegrating Relation

Although spot and forward rates are I(1), they have drifted together over time as Figure 1 shows. If the two interest rates are cointegrated, the cointegrating relation after normalization is of the form

$$s_t = \alpha + \beta f_t + u_t, \tag{32}$$

where α and β are the cointegrating coefficients and u_t's I(0) error terms. Clearly, β is related to μ in (23) by $\beta = 1/\mu$. If the two interest rates are not cointegrated, u_t's are I(1).

We treat spot and forward rates symmetrically and test for cointegration between them using a bivariate VAR(*p*) in Equation (4), which when written explicitly for s_t and f_t is as follows:

$$
\begin{aligned}
s_t &= \mu_1 + \Sigma_{i=1}^{p} \phi_{11}^{(i)} s_{t-i} + \Sigma_{i=1}^{p} \phi_{12}^{(i)} f_{t-i} + \varepsilon_{t,1}, \\
f_t &= \mu_2 + \Sigma_{i=1}^{p} \phi_{21}^{(i)} s_{t-i} + \Sigma_{i=1}^{p} \phi_{22}^{(i)} f_{t-i} + \varepsilon_{t,2},
\end{aligned}
\qquad (33)
$$

where $\varepsilon_{t,1}$ and $\varepsilon_{t,2}$ are bivariate white noise error terms. The normality assumption of the bivariate error process enables us to apply Johansen's (1988,1991) FIML method to (33).

Table 2 presents Johansen's trace statistics for cointegration with VAR(p) models of order $p = 3$, 6, 9 and 12. Of the two statistics presented for each lag, the first is for H_0: $r = 0$ against H_1: $r > 0$ while the second is for H_0: $r \leq 1$ against H_1: $r = 2$, where r is the number of cointegrating relations.

The first test statistic rejects the null of no cointegration in almost all cases except two in the second subsample while the second test statistic does not reject the null of at most one cointegration in all cases. We conclude that the two series are cointegrated.

Table 2 also presents Johansen's FIML estimates of the cointegrating parameters for the full sample as well as for the two subsamples. The FIML estimate of α is negative and significantly different from zero in all cases. This finding is consistent with the presence of a risk premium. More interestingly, the FIML estimates of β are numerically very close to one although they are significantly greater than one in the first subsample. In the second subsample estimates of β are not significantly different from one.

Spot and forward interest rates appear to be cointegrated with a unit coefficient. If they are, the forward premium, defined as $f_t - s_t$, is I(0). Table 3 presents the ADF test statistics for a unit root in the forward premium with the truncation parameters of $k = 2, 5, 8$, and 11 for the three sample periods. The null hypothesis of a unit root is clearly rejected in all cases considered. We conclude that spot and forward rates are cointegrated with a unit coefficient and the forward premium is I(0).

Table 2 Johansen's Cointegration Tests

Hypothesis or Parameters	Order p: 3	6	9	12
Sample period: 1961:1 - 1998:8 (T = 452)				
H_o: r = 0	65.911*	38.063*	34.410*	20.091*
H_o: r # 1	5.319	4.369	5.291	4.764
a	-0.0060*	-0.0057*	-0.0048*	-0.0054*
	(.0011)	(.0013)	(.0013)	(.0016)
ß	1.0393	1.0365	1.0250#	1.0317#
	(.0135)	(.0158)	(.0156)	(.0203)
Sample period: 1961:1 - 1979:12 (T = 228)				
H_o: r = 0	40.416*	23.843*	20.393*	21.521*
H_o: r # 1	1.849	1.895	1.760	2.086
a	-0.0060*	-0.0054*	-0.0055*	-0.0055*
	(.0008)	(.0009)	(.0009)	(.0008)
ß	1.0610	1.0512	1.0541	1.0543
	(.0133)	(.0147)	(.0149)	(.0135)
Sample period: 1983:1 - 1998:8 (T = 188)				
H_o: r = 0	28.385*	22.115*	17.269	10.364
H_o: r # 1	2.346	2.180	2.581	2.396
a	-0.0085*	-0.0082*	-0.0077*	-0.0066*
	(.0024)	(.0023)	(.0024)	(.0032)
ß	1.0520#	1.0459#	1.0415#	1.0274#
	(.0284)	(.0270)	(.0282)	(.0372)

Note: Critical values are 19.96 for the first LR test and 9.24 for the second, respectively. Hamilton (1994, pp. 767 and 768). An asterisk (*) indicates the significance of the statistic at the 0.05 level of significance. A sharp (#) indicates that the ß estimate is not significantly different from 1.

Table 3 ADF Test Statistics for Unit Roots in Forward Premium

	Truncation Parameter *k*			
Variables	2	5	8	11
Sample period: 1961:1 - 1998:8 (T = 452)				
$f_t \& s_t$	-7.743*	-6.308*	-4.786*	-4.049*
Sample period: 1961:1 - 1979:12 (T = 228)				
$f_t \& s_t$	-4.454*	-4.682*	-3.133*	-3.431*
Sample period: 1983:1 - 1998:8 (T = 188)				
$f_t \& s_t$	-4.783*	-3.931*	-2.896*	-2.647

Note: An asterisk (*) indicates the significance of the statistic at a = 0.05 when compared to the asymptotic critical value of -2.868 from Davidson and MacKinnon (1993), p. 708.

4. GMM Estimation of an Error Correction Model

In this section we employ the ECM model of the cointegrated spot and forward rates in Equation (22) and test for the bill market efficiency. Two issues arise when regression analysis is applied to (22). First, standard inference based on linear regression models is not applicable as the cointegrating parameter µis not known. Second, error terms are MA(2) as observations are sampled monthly on three- and six-month Treasury bill yields.

We resolve the first issue of unknown µ by employing a unit cointegrating coefficient as strongly supported by the findings in the preceding section. Given that spot and forward rates are cointegrated with a unit coefficient, the efficient bill market implies two testable hypotheses in (28) and (29). We deal with the second issue of overlapping observations by using Hansen's (1982) GMM estimation. It provides consistent estimates of the parameters in (22) and their asymptotic standard errors when u_{t+3} are serially correlated as well as heteroscedastic.

Table 4 presents the estimation results of an ECM model in (22) with p = 6. For the full sample of 1961:1 to 1998:8 as well as the two subperiods the GMM estimate of ?, the coefficient of $f_t \& s_t$, is positive and the null of ? ' 1 in (28) is not rejected. Table 4 also shows that some of the coefficient estimates for the changes in spot and forward rates are significant in the second subperiod from 1983:1 to

1998:8. The test statistics for the joint null hypothesis in Equation (29) that all c_{ij} coefficients are zero is asymptotically a $?^2$ (12) random variable when p = 6. In the full sample as well as in the two subsamples the test dose not reject the null hypothesis that the coefficients of the changes in spot and forward rates are zero. The findings strongly support that the bill market is efficient.

The intercept term corresponds to the negative expected premium if the risk premium is constant and the bill market is efficient. Its estimate has the expected negative sign in all cases and is significantly different from zero in the second subsample.

5. Concluding Remarks

In this paper we have proposed a new test for market efficiency hypothesis based on a testing equation that is derived from an error correction representation of a bivariate cointegrated system. The test is then applied to test if the Canadian bill market is efficient. The data used are the average yield series of three- and six-month bills at the last auction of each month from January 1961 to August 1998. Two remarkable findings have emerged:

(1) Spot and forward rates are I(1) and cointegrated. Estimation results based on Johansen's FIML method suggest a unit cointegrating coefficient while the ADF test implies that the forward premium is I(0).

(2) The GMM estimation results of an ECM model based on a bivariate cointegrated system of spot and forward rates are consistent with the hypothesis that the bill market is efficient in the full sample as well as in the two subsamples. The results strongly support the hypothesis that Canadian Treasury bill market fully utilizes the information contained in the past spot and forward rates in predicting future spot rates and in setting the forward rates.

Table 4: Parameter Estimates of the VEC Model in Eq. (22)

Variables	1961:1 - 1998:8	1961:1 - 1979:12	1983:1- 1998:8
		Sample Period	
C	-.0017	-.0017	-.0048*
	(.0013)	(.0011)	(.0017)
$f_t \& s_t$	.6076	1.0548#	.9871#
	(.2876)	(.3254)	(.2936)
? s_t	.3770	.4792	.6348*
	(.3117)	(.5162)	(.2664)
? $s_{t\&1}$	.3809	.0608	.6626*
	(.3197)	(.4750)	(.2854)
? $s_{t\&2}$	.1800	-.1127	.2951
	(.3390)	(.3879)	(.2758)
? $s_{t\&3}$	.2222	-.0557	.1778
	(.3215)	(.3779)	(.2697)
? $s_{t\&4}$	.1438	.3034	.2887
	(.2952)	(.4075)	(.1947)
? $s_{t\&5}$	.1922	.5366	.2343
	(.2909)	(.4081)	(.1520)
? f_t	-.0436	-.1168	-.6110*
	(.3075)	(.4537)	(.2967)
? $f_{t\&1}$	-.3186	-.0631	-.6710*
	(.3426)	(.4586)	(.2867)
? $f_{t\&2}$	-.3376	.1417	-.6161*
	(.3535)	(.3561)	(.2644)
? $f_{t\&3}$	-.3598	-.1210	-.4689
	(.3010)	(.3270)	(.2454)
? $f_{t\&4}$	-.2631	-.2666	-.3317
	(.2629)	(.3818)	(.2009)
? $f_{t\&5}$	-.2781	-.5378	-.1892
	(.2115)	(.3716)	(.1670)
T	449	228	185
SEE	.0102	.0065	.0091
R^2	.1141	.2090	.1560
DW	.7082	.7197	.7295
$?^2$ (12)	13.59	10.54	16.60

Note: An asterisk (*) implies statistical significance at a = 0.05.

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