Wide Single-Mode Channels and Directional Coupler by Two-Step Ion-Exchange in Glass

JACQUES ALBERT AND GAR LAM YIP, SENIOR MEMBER, IEEE

Abstract—A simple model for the characterization of single-mode channel waveguides fabricated by a two-step K\(^+\)-Na\(^+\) ion-exchange in glass is described. The model is completely determined with data exclusively for planar waveguide measurements and can be extended to other types of exchange or substrates. A directional coupler was designed, fabricated, and measured to show both the usefulness and accuracy of the model and also the relaxation of the fabrication tolerances provided by the two-step ion-exchange.

I. INTRODUCTION

PASSIVE waveguide components in glass have shown great promise in recent years for various applications needing optical circuitry [1]–[6]. Single-mode polarization-preserving devices in particular are needed whenever the phase information of the incoming lightwave must be conserved upon passage through these devices. Glass provides an ideal substrate for such use because of the good refractive index match with optical fibers. Furthermore, the potassium ion-exchange process in glass yields waveguides with high birefringence, well separated TE and TM modes, and small propagation losses (0.5 dB/cm [1]–[2]). Also, this process is very economical for the batch fabrication of passive devices with highly reproducible characteristics (compared with all-fiber devices made by fusion or grinding and polishing for instance).

In this paper, a new class of single-mode channel optical waveguides made by a two-step ion exchange in glass [4] is described. The purpose of the paper is two-fold. Firstly, a model for the waveguiding channel is developed. This model is based on available data derived from planar waveguide characterizations [7]–[10] and provides, with the help of the effective index method [11], a simple and rapid way to determine fully the dispersion characteristics and field profiles of the modes of these graded-index channels. Secondly, the model is used in the design of two directional coupler structures which have been fabricated and measured to show both the accuracy of the model and the usefulness of the two-step ion-exchange as a means of controlling the performance of waveguiding components fabricated in this manner.

Manuscript received February 4, 1987; revised August 27, 1987. Parts of this paper were presented at the 1986 European Conference on Optical Communication (ECOC '86) in Barcelona, Spain. This work was supported by the NSERC (Canada).

The authors are with the Department of Electrical Engineering, McGill University, Montreal, Quebec, Canada H3A 2A7.

IEEE Log Number 8718509.

One of the advantages of the two-step method is that the fabrication tolerances are relaxed because single-mode waveguides (and their separation in a directional coupler) can be made much wider than with single exchange methods. This will also help in reducing coupling losses to single-mode fibers by relaxing the alignment tolerances.

II. TWO-DIMENSIONAL MODELING OF THE INDEX PROFILE

The ion-exchange process in glass is not a classical diffusion process [12]. Instead of atoms moving into the substrate, ions are exchanged between oxide molecules in the glass and a suitable source of in-diffusing ions (a molten potassium nitrate bath in our case). The exchange must be on a one-to-one basis (for ions of equal valence) to preserve the charge neutrality of the glass. Also, local charge imbalances occur during the process. These two effects lead to a concentration-dependent diffusion coefficient which depends on the self-diffusion coefficients of the ions participating in the exchange.

The equation governing the evolution of the concentration of new ions in the glass takes the following form [13]–[14]:

\[
\frac{\partial c}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{D_1}{1 - \alpha c} \frac{\partial c}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{D_1}{1 - \alpha c} \frac{\partial c}{\partial y}\right]
\]

(1)

with

\[ c = c_1/c_0 \]

and

\[ \alpha = 1 - D_1/D_0 \]

where \( c_1 \) and \( D_1 \) are the concentration and self-diffusion coefficient of the incoming ions, \( D_0 \) is the self-diffusion coefficient of the outgoing ions, and \( c_0 \) is the concentration of outgoing ions present in the glass prior to the exchange. Diffusion is restricted to the \( x-y \) cross-sectional plane of the waveguiding structures and \( t \) is the exchange time.

In order to solve (1), the values of the numerical parameters \( D_1 \) or \( \alpha \) (or equivalently \( D_0 \)) must be found. For the exchange of potassium \((K^+)\) and sodium \((Na^+)\) ions in soda-lime glass at a temperature of 374°C, it has been established that the much smaller \( Na^+ \) ions have a higher mobility than the \( K^+ \) ions yielding a ratio \((D_1/D_0)\) of 1/500, which corresponds to \( \alpha = 0.998 \) [12]. We are going to use this available data even though our working...
temperature is a little higher at 385°C. The value of $D_1$ can be calculated from our previous characterization of the planar $K^+$-ion exchange in soda-lime glass [9]. In that work it was shown that the one-dimensional (1D) profile of refractive index change (obtained from mode index measurements), and hence of replaced ion concentration (assuming proportionality between the two), is well described by the following Gaussian function:

$$c(x) = c(0) \exp\left(-\frac{x^2}{4a^2D_1t}\right)$$

with $x$ being the depth coordinate measured from the surface of the substrate and $D_1$ the 'effective diffusion coefficient' (equal to $10.7 \times 10^{-16} \text{ m}^2/\text{s}$ for $T = 385°C$). On the other hand, we have also shown [15] that an exact numerical solution of the 1D equivalent of (1) can be fitted with good accuracy by a Gaussian function of the form:

$$c(x) = c(0) \exp\left(-\frac{x^2}{4a^2D_1t}\right)$$

with

$$a = 1.17.$$ 

Therefore, $D_1$ is obtained by combining (2) and (3) as:

$$D_1 = D_e/4(1.17)^2 = 1.95 \times 10^{-16} \text{ m}^2/\text{s}$$

(at $T = 385°C$).

Finally, suitable boundary and initial conditions must be specified to solve (1) unambiguously. The ion-exchange considered here consists of two steps: a first exchange through a finite opening to define guiding channels and a second exchange over the whole plane of the substrate to adjust the lateral waveguiding properties of the channels by modifying the effective index of the surrounding areas (as will be described in the next section). Initially $c = 0$ everywhere. At $t = 0$, the concentration at the surface $c(0, y)$ takes a finite, constant value in the unmasked areas and remains zero elsewhere. The concentration at the other boundaries at $x = 0$ and $y = \infty$ remains fixed at zero for all times.

It should be noted that $c(0, y)$ is not equal to 1 for $K^+-Na^+$ exchange even though $c$ is normalized with respect to $c_0$. This is because the exchange of these two ions is limited to approximately 90 percent of the full Na$^+$ content of the glass [12], again probably due to the size difference of the ions. The renormalization of (1) to have $c(0) = 1$ leads to an effective value for $a$ of 0.898 [15].

During the second step of exchange the boundary condition at $x = 0$ becomes $c(0, y) = 1$ for all $y$.

Under these conditions, (1) has been solved by an iterative finite-difference numerical method (similar to the one used in [14]) for a variety of diffusion times and mask apertures. For the purposes of the present work, however, the aperture width was fixed at 10 μm to show the possibility of single-mode operation with large lateral dimensions (relative to a wavelength of 0.633 μm) and to permit the use of conventional photolithographic techniques with good reproducibility of the results. Also, this size is ideal for single-mode use at the longer wavelengths preferred in communication systems (1.3 and 1.55 μm) because it becomes compatible with the core dimensions of the fibers. Although the experiment was performed at 0.633 μm for convenience, the device should remain single-moded at 1.3 or 1.55 μm.

The waveguiding properties of the channels are modified by changing the first (masked) exchange time $t_1$ and second (planar) exchange time $t_0$, but keeping the total time equal to 1 hr. Consequently, the center of the channel has always the same diffusion depth of 1.96 μm and supports a single, well-confined depth mode of each polarization (TE and TM) [9]. The areas that were masked during $t_1$ will have a depth that depends on $t_0$, varying from 0 to 1.96 μm and allowing for lateral mode control.

Fig. 1 shows the geometry of the calculation grid and the concentration contours resulting from two values of $t_1$. It is interesting to note that the fairly large control over the waveguiding properties offered by this two-step method is obtained without much increase in the complexity of the fabrication procedure. The only additional step is an immersion of the whole substrate (after removal of the mask) in the salt melt, without new high-vacuum or photolithography steps.

III. Waveguiding Properties of Two-Step Ion-Exchanged Channels

For the waveguides considered here, the difference between the surface index $n_s$ and the substrate index $n_b$ is fairly small: $10.8 \times 10^{-3}$ for TM modes and $8.8 \times 10^{-3}$ for TE modes [9]. In the substrate, the strongest index gradient occurs in the depth ($x$) direction. Taking a typical depth of 1 μm and $\Delta n$ of $1 \times 10^{-2}$ we get approximately $\Delta n/\Delta x = 1 \times 10^4$ which is much smaller than the free-space wavenumber of the light used in the experiments (He-Ne laser, $\lambda = 0.6328$ μm, $k_0 = 1 \times 10^5$), and the scalar wave approximation can be used [16]. Then, the channels will support quasi-TE modes obeying:

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + (k_0^2 n_s^2(x, y) - \beta^2) E_y = 0$$

and quasi-TM modes obeying:

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (k_0^2 n_s^2(x, y) - \beta^2) H_y = 0$$

with the assumption that

$$n(x, y) = n_b + (n_s(TE) - n_b) \times c(x, y),$$

for TE modes.

and

$$n(x, y) = n_b + (n_s(TM) - n_b) \times c(x, y),$$

for TM modes.

Since $n_s^2(x, y) \neq n_b^2(x) + n_b^2(y)$, equation (4) (or (5)) is not formally separable [17]. Therefore, the full 2D problem should be tackled to obtain an exact solution.
Due to the complicated nature of the index profile, such a solution can only be obtained by numerical computation. Various methods exist for this task but the computational cost is high, rendering them impractical in a design situation where various approaches to a given problem have to be tried and evaluated quickly. An elegant way to overcome this problem is provided by the so-called "effective index method" which reduces the 2D problem to two simpler 1D problems. This method involves replacing \( E'(x, y) \) (or \( H'(x, y) \)) by a product of two functions in the following manner:

\[
E'(x, y) = F(x, y) G(y).
\]

Then (4) becomes:

\[
\frac{G\partial^2 F}{\partial x^2} + \frac{G\partial^2 F}{\partial y^2} + \frac{2\partial F}{\partial y} \frac{\partial G}{\partial y} + (k_0^2 n^2(x, y) - \beta^2) FG = 0. \tag{9}
\]

We get

\[
\frac{\partial^2 F}{\partial x^2} + (k_0^2 n^2(x, y) - \beta^2) FG = 0. \tag{10}
\]

Equation (10) can be solved using any of the many methods available to solve planar graded-index waveguide problems [18]–[20]. The local effective index \( N_{\text{eff}}(y) \) (along with \( F(x, y) \)) is then found for all the values of \( y \) and substituted in (8) (also a 1D waveguide problem) to give \( G(y) \) and the propagation constant \( \beta \).

This fairly detailed description of the well known effective index method was made to emphasize its appropriateness in the case discussed here. First, as in the case of strip-loaded planar guides [21], the effective index \( N_{\text{eff}} \) is unambiguously defined at all lateral points \( y \) because of the shallower planar guide surrounding the channel. Second, it will be shown shortly that even for fairly large \( t_1 \) (i.e., planar guide much shallower than the channel), the variation of \( F(x, y) \) with \( y \) is truly negligible, making the method almost approximation-free.

Since \( n(x, y) \) represents a graded-index profile, \( N_{\text{eff}}(y) \) is a continuous function of \( y \) and we should solve (10) at an infinite number of points to describe it completely for use in (8). This is of course impossible and instead we will derive an analytical model for \( N_{\text{eff}}(y) \) valid for a practical range of values of \( t_1 \). This model allows for the solution of waveguiding structures defined by any combination of apertures in \( y \) for any set of fabrication conditions, thereby easing the design process considerably.

The depth characteristics of the model will be derived only once so it is worthwhile to use an exact numerical method, even if fairly lengthy, to solve (10). The method chosen is the Rayleigh–Ritz variational procedure with Hermite-Gaussian basis functions [22]–[23]. The eigenvalues \( N_{\text{eff}}(y) \) of (10) are obtained by finding the field \( \Psi(x) \) (expressed as a linear combination of basis functions):

\[
\Psi(x) = \sum_{i=1}^{M} c_i \Phi_i(x)
\]

which maximizes the functional:

\[
N_{\text{eff}}(y_0) = \max_{\Psi} \int_{-\infty}^{\infty} dx \left( \int_{-\infty}^{\infty} dx \Psi^2 - \frac{\nabla \Psi^2}{k_0^2} \right) \int_{-\infty}^{\infty} dx \Psi^2 \tag{11}
\]
This leads to a matrix eigenvalue problem of dimension \( M \), the number of basis functions used. Since accuracy improves with increasing \( M \), we made several runs and established that \( N_{\text{eff}} \) is correct to one part in \( 10^2 \) for \( M = 21 \). Now, \( n(x, y) \) as described in Section II is only defined on a discrete grid in the \( x, y \) plane. Therefore, (10) was solved along \( x \) for each \( y \)-grid value and the integrals involving \( n(x, y) \) in (11) were computed numerically by interpolation between the \( x \)-grid values. This calculation was made for 6 values of \( t_1 \) (keeping the total time to 1 hr) for each polarization (TE and TM). Then the sets of values of \( N_{\text{eff}} (y) \) obtained in each case were fitted (by a least-squares method) with the following function which represents the exact solution of a classical (Fickian) diffusion process (and which turns out to be a pretty good approximation of this more complicated case):

\[
N_{\text{eff}} (y) = N(\infty) + \frac{(N(0) - N(\infty))}{2} \cdot \frac{\operatorname{erf} \left( \frac{y + D/2}{H} \right) - \operatorname{erf} \left( \frac{y - D/2}{H} \right)}{H}
\]  

(12)

where

- \( H \) fitting parameter,
- \( N(0) \) effective index of a planar guide exchanged for \( 1 \) h (corresponds to \( N_{\text{eff}} (y = 0) \)),
- \( N(\infty) \) effective index of a planar guide exchanged for time \( t_0 \) (i.e., \( N_{\text{eff}} (y) \), away from the edge of the mask),
- \( D \) width of the aperture in the mask.

The results for \( H \) are summarized in Table I. These \( H \) values were found by a purely statistical method but it is interesting to compare them with a value derived from a physical argument. As was already noted, (12) is the exact solution of a classical 1D diffusion process in which a finite quantity of material diffuses symmetrically from an initial constant concentration layer of width \( D \) [24]. In that case, the value of \( H \) is determined from an effective diffusion coefficient in the same manner as in formula (2) by:

\[
H = \sqrt{D t_1}.
\]  

(13)

Now, glass is an isotropic material and the diffusion characteristics should be independent of direction. Therefore, we should be able to take \( D_x = D_y \). For the duration of the side diffusion to be used in (13), we will take \( t_0 \) since the presence of a metallic mask inhibits the exchange of ions near the surface of the substrate during \( t_1 \). This becomes a poorer estimate when \( t_1 \) becomes large because then the exchange has reached greater depths where side diffusion can occur.

Theoretical estimates have been calculated for \( H \) by using (13) and values of \( D_x \) from [9]. The results, presented in Table I, show an excellent agreement with the fitted values. It may be useful to remark at this point that this model for single-mode channel waveguides made by two-step ion-exchange is completely determined from measurements on planar waveguides (i.e., \( N(0), N(\infty), \text{and} \; D_x \) and, therefore, may be applied to all types of exchange for which such planar waveguide studies are available [7]-[10].

Also, Fig. 2 presents \( F(x, 0) \) along with \( F(x, \infty) \) for the fairly extreme case of \( t_1 = t_0 = 0.5 \) h. We can see that indeed \( dF/dy \) is very small and will be even smaller for lower values of \( t_1 \) where the waveguides support only one lateral mode (as we will see later). Using these results, along with (12), in (8) allows for the calculation of the dispersion curves, which can be plotted in normalized form with:

\[
b = \left( \frac{\beta}{k_0} \right)^2 - N^2(\infty), \quad V = k_0 D \sqrt{N^2(0) - N^2(\infty)}.
\]

(14)

The method used to generate the \( \beta \) values from (8) is of the variational type but with a single trial function as described by Sharma et al. [25]. The choice of this method is motivated by its ease of use, involving a single root-search, and also because it was found to be very accurate in the range of validity of our model. The method uses a Gaussian trial function for \( G(y) \):

\[
G(y) = \exp \left( -\sigma y^2 \right)
\]

(15)

with a single adjustable parameter \( \sigma \), which is found by maximizing a functional similar to (11). The appropriateness of this form of trial function can be justified by the fact that a Rayleigh–Ritz procedure was also tried for comparison purposes and that the desired accuracy was already reached with a single trial function (\( M = 1 \)): the fundamental Hermite–Gaussian, which turns out to be a pure Gaussian. We also tried the WKB dispersion relation with two turning points and found reasonable, although less accurate, agreement. This is seen in Fig. 3 which shows these dispersion curves with \( N(0) = 1.51548 \) for TE modes and \( N(0) = 1.51644 \) for TM modes (with \( n_b = 1.513 \)). It is seen that the channels support a single mode of each polarization when \( V \) is less than 3.5, cor-
TABLE I

<table>
<thead>
<tr>
<th>$t_1$ (min)</th>
<th>$H(TE)$</th>
<th>$H(TM)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fit $H$ ($\mu m$)</td>
<td>Eq. (13) $H$ ($\mu m$)</td>
</tr>
<tr>
<td>4</td>
<td>1.77</td>
<td>1.91</td>
</tr>
<tr>
<td>6</td>
<td>1.80</td>
<td>1.87</td>
</tr>
<tr>
<td>9</td>
<td>1.80</td>
<td>1.82</td>
</tr>
<tr>
<td>12</td>
<td>1.79</td>
<td>1.77</td>
</tr>
<tr>
<td>17</td>
<td>1.72</td>
<td>1.67</td>
</tr>
<tr>
<td>30</td>
<td>1.54</td>
<td>1.40</td>
</tr>
</tbody>
</table>

averages for $t_1$ between 6 and 9 minutes:

- $<1.80 \pm 0.01$
- $<1.82 \pm 0.05$
- $<1.76 \pm 0.07$
- $<1.80 \pm 0.05$

- $q_{TE} = 10.82 \times 10^{-14}$ m$^2$/s
- $q_{TM} = 10.61 \times 10^{-14}$ m$^2$/s

responding to $t_1$ less than 10 min, keeping all other parameters fixed.

Without much loss in accuracy the usefulness of the model can be improved further by the use of a single value for $H$ in the well-guided single-mode range, taken to be the average of the values found for $t_1 = 6$ to 12 min. With this modification, Fig. 4 shows the exact $N_{eff}$ along with the model fit for a few values of $t_1$. The difference in $\beta/k_0$ resulting from using the average $H$ instead of the exact one (from either the fit or formula (13)) amounted to much less than one part in 10$^5$.

**IV. DIRECTIONAL COUPLER DESIGN**

The approach taken to analyze the directional coupler (DC) will be to calculate the normal modes of the structure consisting of two parallel channels [26]. Then, by projecting the optical field coming in from one branch onto the symmetric and antisymmetric modes of the coupler, the output characteristics will be obtained by propagating the two modes thus launched with their respective propagation constants down the length of the coupling region. The interference between the modes causes the power to oscillate between the two channels until their separation increases again to a value for which there is no more coupling. The power in each channel remains constant from that point on and can be calculated.

The normal modes of the DC are found by solving (8) with the following index profile (with $N(y)$ defined by (12)):

$$N_{eff}(y) = N(y + W/2) + N(y - W/2) - N(\infty)$$

(16)
and trial function [25]:

\[ G_e(y) = \exp \left( -\sigma_e(y + W/2)^3 \right) \]

\[ + s \exp \left( -\sigma_e(y - W/2)^3 \right) \]

(17)

with \( s = +1 \) for the even mode (with \( \sigma_e \)), \( s = -1 \) for the odd mode (with \( \sigma_o \)), and \( W \) the center to center spacing between the channels.

Examples of the modes thus obtained are shown in Fig. 5 (where the polarization labeling refers to the plane of the substrate). As will be seen below, the parameter of importance is

\[ \Delta \beta = \beta_e - \beta_o \]

(18)

the difference between the propagation constants of the even and odd modes. This quantity is plotted in Fig. 6 for various values of the separation distance \( W \). In this type of analysis, it is the interference between the even and odd modes that produces a periodic power transfer between the channels. Therefore, the problem is reduced to finding out the total relative phase shift accumulated along the length of the DC and the relative amplitudes of the two modes at the start of the coupling region. At \( z = 0 \), we have the initial condition

\[ \Psi(y, z = 0) = A_e G_e(y) + A_o G_o(y) \]

(19)

and, further down at any point \( z \), the modes will have propagated independently and the field becomes

\[ \Psi(y, z) = A_e G_e(y) \exp(-j\beta_e z) \]

\[ + A_o G_o(y) \exp(-j\beta_o z) \]

(20)

In terms of amplitude, (20) can be rewritten as

\[ |\Psi(y, z)| = A_e |G_e(y) + R G_o(y) \exp(+j\Delta \beta z)| \]

(21)

with

\[ R = A_o / A_e \]

(22)

In a real device, the separation distance \( W \) must be varied to permit a narrow (a few micrometers) coupling section connected to widely separated input–output sections to facilitate butt coupling to adjacent waveguides (such as optical fibers with an outer diameter of more than 100 \( \mu m \)). Thus, \( W \) is a function of \( z \) which implies that \( \Delta \beta \) is also a function of \( z \). The accumulated phase difference becomes [27]:

\[ \Delta \Phi(z) = \int_0^z \Delta \beta(z) \, dz \]

(23)

The functional dependence of \( \Delta \beta \) on \( z \) is found by using the relationship between \( W \) and \( z \) and by fitting straight lines (again by a least-squares procedure) to the data presented in Fig. 6 for \( \log_{10}(\Delta \beta/k_0) \) versus \( W \). The results of these fits is given in Table II in the following form:

\[ \Delta \beta/k_0 = A \exp(-mW) \]

It is to be noted that this approach to the DC problem with variable spacing neglects both phase-front curvature and radiation loss due to the nonuniformity in the axial direction (see, for example, [28]). However, the actual designs will have bending angles of less than 0.5°, which should minimize these deleterious effects. Fig. 7 shows the two designs that will be used in this study. In both cases, light is launched as a single mode of channel \( a \) and the cross-power transfer is defined as:

\[ \eta = P_b / (P_a + P_b) \]

(24)

where \( P_a \) and \( P_b \) are the powers measured at the outputs of channels \( a \) and \( b \). A theoretical estimate of \( \eta \) is ob-
Fig. 6. Propagation constant difference between the even and odd modes of the DC as a function of center-to-center spacing for two values of $t_1$. 

Fig. 7. Actual designs of the couplers with main dimensions included but not drawn to scale.

We see that complete transfer is only possible when $R = 1$, i.e., for equal excitation of the even and odd mode at $z = 0$. Because of (25), this condition is fulfilled for design 1 when the incident field is $G_a$ or $G_o$. For design 2, it is necessary to calculate the projection of the incident field $G_a(y)$ on each normal mode of the DC to get $A_e$ and $A_o$ and $R$ because the channels are strongly coupled at that point. The calculation is as follows:

\[ R = \frac{\sqrt{\frac{\sigma_e}{\sigma_o}} (\sqrt{\frac{\sigma_o}{\sigma_e} + 1} - 1 - \exp (-W^2 \sigma_e / (\sigma_e + \sigma_o)))}{(1 + \exp (-W^2 \sigma_e / (\sigma_e + \sigma_o)))^{1/2}} \]

where $\sigma_e$ and $\sigma_o$ refer to the parameters of the channel, even, and odd modes found in the variational procedure of Section III for the given values of $t_1$ and $W$.

V. ACTUAL MASK DESIGN

The central parallel coupling region was chosen to be $L = 10$ mm for both designs, with a center to center spacing $W_0$ of 16 $\mu$m. These conditions correspond to approximately one coupling length for the parallel section at the values of $t_1$ of interest. The angles of the discrete bends were limited to be less than 0.5° to minimize in-plane scattering (even at $t_1 = 9$ min the difference in effective indices between the center of the channel and its surroundings is only $3.9 \times 10^{-4}$ for TE and $4.6 \times 10^{-4}$ for TM). In both designs the variation of $W$ with $z$ is linear of the form:

\[ W(z') = W_0 + pz' \]

where $z'$ measured from the start of the transition. The first tapering section on each side of the parallel section brings $W$ from 16 to 46 $\mu$m over a length $l_1 = 1.75$ mm for design 1. For design 2, branch a does not bend. The relative angle between the branches is thus reduced by half and the tapered length $l_2$ to reach the same final separation of $W = 46$ $\mu$m is 3.5 mm. These choices were made so that both designs would have the same transfer characteristics as will be seen below. After the first tapering sections, successive bends bring the final value of $W$ to 300 $\mu$m for design 1 and to 200 $\mu$m for design 2 in a total length of 40 mm.

While design 1 is in principle preferable because of the total symmetry of operation from any of the four ports and
also because a total transfer is possible ($R = 1$), we will see that problems do arise because of the multiple bends and the inherent weak guidance of the channels resulting from the two-step ion-exchange. Design 2 was prepared and included on the same mask to see if a similar structure with fewer bends would perform better. In that case the scattering from branch $a$ is kept to a strict minimum by making it a straight channel. No input section is included with branch $b$ because the scattering problems mentioned above are thought to affect $\eta$ primarily from the input section bends.

Now equipped with the full design, we can calculate $R$ and $\Delta \Phi$:

$$\Delta \Phi_1 = \int_0^{t_i} k_0 A \exp (-mW(z')) (-dz') + \Delta \beta(W_0) L$$

$$+ \int_0^{t_i} k_0 A \exp (-mW(z')) (dz')$$

$$\Delta \Phi_2 = \Delta \beta(W_0) L + \int_0^{t_i} k_0 A \exp (-mW(z')) \, dz'$$

$$\equiv \Delta \beta(W_0)(L + 1/mp_2)$$

with

$$p_1 = (46 - 16) \, \mu m / 1.75 \, \text{mm} = 17.14 \, \mu m / \text{mm}$$

$$p_2 = (46 - 16) \, \mu m / 3.50 \, \text{mm} = p_1 / 2$$

so that $\Delta \Phi_1 = \Delta \Phi_2$ are functions of $\Delta \beta(16 \, \mu m)$ and of $m$ only. Typical results are shown in Table II along with the values of $R$ calculated from (28). Equation (27) can now be solved for various experimental conditions. Fig. 8 shows the result of such calculations, where $\eta(\text{TE})$ is plotted against the length of the center section $L$ for two values of $t_i$, keeping all other variables fixed (including $W_0$ and the shape of the transition sections). This shows the fairly wide range of cross-power transfers that the two-step ion-exchange allows with a single fabrication mask, i.e., for a given length $L$ (for example $\eta$ can go from 60
to 95 percent for \( L = 11.5 \) mm in the single-mode range of the channels.

**VI. Fabrication and Measurement**

The substrates used in the experiments are ordinary soda-lime microscope slides (available from Fisher Scientific Co.). We have made an extensive characterization of the planar waveguides fabricated in these substrates by ion-exchange in a molten potassium nitrate (KNO₃) bath, and they were found to yield waveguides with good and reproducible characteristics [9]. The fabrication procedures for two-step ion-exchange are outlined in Fig. 9. After cleaning, a layer of aluminium (Al) is deposited by vacuum evaporation. Then a photoresist layer is applied and exposed through a suitable mask to define the channels. After the development of the resist, the Al is removed from the exposed areas by liquid etching, leaving clear channels in the Al-covered substrate. This structure is dipped in the molten KNO₃ bath maintained at 385°C in a thermocouple-controlled furnace. After an exchange time \( t_1 \), the substrate is cleaned, the remaining Al removed, and the second exchange is performed for time \( t_2 \) until \( t_1 + t_2 = 1 \) h.

Coupling of the light in and out of the waveguide is achieved through flint glass prism-couplers on a precision goniometer. Since the \( m \)-lines of adjacent channels would overlap, the near-field of light tunneling through the base of the prism is observed with a microscope and projected onto a screen, as shown in Fig. 10. The horizontal extension of the light spots on the screen is due to the fact that light escapes gradually from the channels into the prisms. The vertical extension of the spots represents the lateral mode profile of the channels. Actual measurements were performed by scanning a large area photodetector partially obstructed by a narrow long slit vertically across the output pattern. Because of the slit, the detector integrates horizontally all the light coming out of the channels as a function of \( y \)-position (vertical in the figure). Typical results are shown in Figs. 11 and 12. This configuration allows for the visualization of the in-plane scattering, also output by the prism.

Numerically, the measured \( \eta \) are obtained from the traces by using the peak amplitudes. This assumes that the fields of the two channels have the same shape (i.e., a Gaussian with the same width) since they result from exactly the same fabrication. This means that the power in each channel is given by the amplitude of the trace (the detector is a POWER meter) multiplied by a factor which is the same for both channels and cancels out in (24).

**VII. Results and Discussions**

The experiments were carried out using the mask described in Section V for various values of \( t_1 \). For design 1, four measurements of \( \eta \) are possible, corresponding to input from the two branches in both polarizations. Theoretically, since the device is symmetric, the results should be independent of which branch is chosen as the input, even when scattering and absorption losses are considered. This was not found to be the case as shown in Table III. It is believed that this problem arises because of differences in the launching conditions from the two branches. The input coupling being nonperfect, some of the light not coupled into the channel propagates in the surrounding planar guide and can be captured by the waveguiding structure further down at one of the various discontinuities in the pattern. Since it is impossible to
couple into each branch exactly identically, this effect occurs in a quasi-random manner and independently for $P_a$ and $P_b$, completely spoiling the measured value of $\eta$. To support this explanation, the traces corresponding to design 1 showed generally more scattering as indicated in Fig. 11. As was said earlier, design 2 was chosen to alleviate this problem by minimizing the number of bends of the whole structure and eliminating them completely from the input section.

A few traces of the results obtained with design 2, showing much less scattering, and nice Gaussian-like lateral modes, are presented in Fig. 12.

With this design, the validity of the model for the channels can be verified with more confidence and a comparison between theoretical and experimental values of $\eta$ is made in Fig. 13 as a function of $t_1$ for TM modes. The agreement is remarkable when one remembers that the theoretical curve is determined with data for planar ion-exchange measurements exclusively. In particular, the position of the peak efficiency is very accurate.

The drop in the experimental efficiency for $t_1$ below 9 minutes is believed to arise from a loss of confinement at the bends when $\Delta N_{eff}$ becomes smaller than some threshold value (in fact, for $t_1 = 5$ min, $\Delta N_{eff} = 2.4 \times 10^{-4}$ for both TE and TM and even at $0.5^\circ$ the bend angles may be too large). This confinement problem indicates

---

**TABLE III**

<table>
<thead>
<tr>
<th>sample</th>
<th>$t_1$</th>
<th>TE</th>
<th></th>
<th>TM</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Exp.</td>
<td>Eq. (27)</td>
<td>Exp.</td>
<td>Eq. (27)</td>
</tr>
<tr>
<td></td>
<td>input $a$</td>
<td>$b$</td>
<td>$a+b$</td>
<td>input $a$</td>
<td>$b$</td>
</tr>
<tr>
<td>11</td>
<td>22'</td>
<td>.45</td>
<td>.48</td>
<td>.92</td>
<td>.40</td>
</tr>
<tr>
<td>19</td>
<td>6'</td>
<td>.59</td>
<td>.59</td>
<td>.88</td>
<td>.65</td>
</tr>
<tr>
<td>20</td>
<td>5'</td>
<td>.45</td>
<td>.43</td>
<td>.80</td>
<td>.41</td>
</tr>
<tr>
<td>21</td>
<td>7.5'</td>
<td>.59</td>
<td>.44</td>
<td>.97</td>
<td>.67</td>
</tr>
</tbody>
</table>
that structures made with the two-step ion-exchange method should be designed to operate as close as possible to the second mode cut-off to reduce the scattering losses by increasing the normalized frequency \( V \).

For \( t_1 > 10 \) min the channels are no longer single mode and the measured efficiencies depend somewhat on the launching conditions, i.e., on which modes are excited at the start. The theoretical curve of Fig. 13 drops at high values of \( t_1 \) because it is based on coupling of the fundamental mode of the channels which become more tightly confined as \( t_1 \) increases. Experimentally, however, values of transfer approaching 100 percent could be achieved at \( t_1 = 13 \) and 17 min (see Fig. 12) by varying the angle of incidence and width of the laser beam incident on the input prism-coupler. Therefore, one should design a structure which, for a given desired value of \( \eta \), would operate as close as possible to the second mode cutoff by adjusting \( t_1 \) accordingly. The design procedure itself is very simple and can be summarized as follows: 1) from a desired value of \( \eta \) compute \( \Delta \Phi \) from (27), 2) choose the right combination of \( L, W_0 \), and \( p \) from (31) or (32) while using the \( m \) and \( \Delta \Phi \) values corresponding to the preferred \( t_1 \) as shown in Table II.

VIII. Conclusion

A simple analytical model for channel waveguides fabricated by a two-step ion-exchange process in glass based exclusively on experimental data for planar ion-exchange characteristics was presented. This model is believed to be applicable regardless of the ion-exchange species and of the substrates used provided that suitable adjustments are made to account for the different material properties. The validity of the model was tested by applying it to a case of evanescent coupling between two parallel channels followed by nonparallel transition sections. Excellent agreement was found within certain limits of applicability.

The novelty of this approach rests in the use of the effective diffusion coefficients and approximate fitting functions which reduce the solutions of the nonlinear diffusion equations of ion-exchange to the more familiar results of classical diffusion theory. Once this simplification is achieved, and with the help of the effective index method, the waveguiding properties of the channels can be obtained quickly and easily.

Finally, the process of two-step ion-exchange itself was shown to be a useful method to fabricate passive waveguide components in glass. This is because a given design (always subject to some inaccuracies due to approximations in the design formulas or to uncertainties in the fabrication parameters) can be fine-tuned to the desired performance simply by adjusting the durations of the two exchanges involved without any additional fabrication steps. Also, single-mode waveguides of fairly wide dimensions (dimensionless width \( k_0D = 100 \)) can be fabricated with this method by controlling the effective index of the planar waveguide which acts as a cladding for the graded-index channel. This easy control of the lateral waveguiding and confinement properties might prove useful in devices where interactions take place by perturbations of the evanescent fields which propagate outside of the core of the waveguides such as sensor systems and waveguides with nonlinear claddings or substrates.

References

Jacques Albert was born in Rimouski (Canada), on October 20, 1956. He received the B.Sc. degree in physics from the Université de Montréal in 1978, and the M.Sc. degree in physics from the Laboratoire de Recherches en Optique et Laser, Université Laval (Québec, Canada), in 1980.

From 1981 to 1983 he was with the Atmospheric Environment Service, Environment Canada. In 1983, he joined the Department of Electrical Engineering, McGill University, to pursue work towards the Ph.D. degree.

Mr. Albert is a member of the Optical Society of America and of the Canadian Association of Physicists.

Gar Lam Yip (S’63–M’67–SM’75) was born in Shanghai, China. He received the B.Sc.(Hon.) degree in 1960 from Imperial College, London University, England, the M.Sc. degree in 1963 from Queen’s University and the Ph.D. degree in 1967 from the University of Toronto, Canada, all in electrical engineering.

Currently, he is a Professor of Electrical Engineering at McGill University, where he has been teaching and doing research in electromagnetic waves and optics, microwaves, and optical waveguides since 1967. In 1969, he initiated research on fiber and integrated optics at McGill. He was the Technical Program Chairman for the North American Radio Science (URSI) Meeting held at Quebec City in June, 1980, and organized, among others, Special Sessions in Fiber Optical Communications, culminating in a “Special Issue in Optical Communications,” July–August 1981 of Radio Science, of which he was Guest Editor. In March 1981, he spent one month in China as a U.N.D.P senior expert in fiber communications giving a course called “Introduction to Optical Waveguides” and seminars on fiber and integrated optics.

Dr. Yip was Chairman of the IEEE Montreal Joint Chapter on AP-S/MTT-S from 1975 to 1978 and also since 1985.