

## **Heterogeneity in Expectations, Risk Tolerance, and Household Stock Shares**

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This paper investigates the relationship between stock share and expectations and risk preferences using linked survey responses and administrative records from account holders. The survey allows individual-level, quantitative estimates of risk tolerance and of the perceived mean and variance of stock returns. Estimated risk tolerance, expected return, and perceived risk have economically and statistically significant explanatory power for the distribution of stock shares. Relative to each other, the magnitudes are in proportion with the predictions of benchmark theories, but they are all substantially attenuated. MBA graduates have more stable beliefs, more knowledge about their account holdings, and less attenuation.

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The source of heterogeneity in portfolio choices is an important question for household finance (Campbell, 2006). Theories, such as consumption CAPM, predict that the share of risky assets should be positively related to their expected returns, negatively related to their risk, and positively related to investors' risk tolerance. Heterogeneity in preferences and beliefs are therefore natural candidates for explaining heterogeneity in household portfolios.

The existing literature addresses various aspects of heterogeneity. Differences in experience can cause different portfolio choices of households (Malmendier and Nagel, 2011; Seru, Shumway and Stoffman, 2010) as well as professional investors (Vissing-Jorgensen, 2003; Greenwood and Nagel, 2009). These effects may operate through preferences and beliefs as experience can influence both.<sup>1</sup> The association of portfolio allocation with wealth, and individual heterogeneity in that association, may also be driven, at least in part, by expectations and preferences (Calvet and Sodini, 2014). The recent literature has established the role of risk preference and beliefs about future returns in the stock share of household portfolios, although most papers examined stock ownership, that is, the extensive margin, because there are frequently so many households with no stock holdings in the data. The focus on the extensive margin makes quantitative comparisons against benchmark portfolio choice models difficult because their predictions largely concern the magnitude of the intensive margin response of portfolio shares to preferences and beliefs.<sup>2</sup> Indeed, explaining heterogeneity at the intensive

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<sup>1</sup> See, for example, Vissing-Jorgensen (2003), Glaser and Weber (2005), Hurd, van Rooij and Winter (2011), Hudomiet, Kezdi and Willis (2011), Amromin and Sharpe (2012), Hoffman, Post and Pennings (2013), and Guiso, Sapienza and Zingales (2014). Heterogeneity in expectations is also important in other contexts, for example, for the housing market (see Piazzesi and Schneider, 2009) and for inflation (see Malmendier and Nagel, forthcoming; Armantier et al., 2013; Bruine de Bruin et al., 2011).

<sup>2</sup> In papers that examine the extensive margin, Barsky, Juster, Kimball and Shapiro (1997), Dohmen, Falk, Huffman and Sunde (2010), Dohmen, Falk, Huffman, Sunde, Schupp, and

margin, that is, the share of stocks in the portfolio of stock market participants, has remained rather elusive (Brunnermeier and Nagel, 2008). An important reason behind the scarcity of empirical results is the lack of appropriate data. Good data on portfolio composition are needed for a large enough sample of stockholding households, combined with appropriate measures of preferences and beliefs.

This paper takes a comprehensive approach to examining heterogeneity in portfolio choice by using a distinctive data set created by the Vanguard Research Initiative (VRI) that combines administrative account data and survey responses for a large sample of Vanguard account holders. The VRI has multiple features that make it especially well-suited for examination of heterogeneity in stock holdings.

First, it is a large sample of stock holders. Moreover, despite being drawn from the account holders of a single company, the characteristics of the sample are broadly representative of the targeted population of households with non-negligible financial assets. Hence, unlike most studies that focus on the extensive margin for stock holdings, this sample will allow for meaningful inferences about the intensive margin of portfolio choice.

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Wagner (2011) and Guiso, Sapienza and Zingales (2014) show that more risk tolerant individuals are more likely to hold stocks, while Dominitz and Manski (2007) and Hurd, Rooij and Winter (2011) show that individuals with higher levels of stock market expectations and lower perceived risk are more likely to hold stocks. Kimball, Sahm and Shapiro (2008) model the intensive margin. Kezdi and Willis (2011) and Dimmock, Kouwenberg, Mitchell and Peijnenburg (2013) combine the extensive and intensive margins in Tobit-type models and establish associations with risk tolerance, expectations and ambiguity aversion, respectively. Vissing-Jorgensen (2003) and Amromin and Sharpe (2012) show that expectations are related to the share of stocks among stockholders but they do not consider risk tolerance. Weber, Weber and Nasic (2013) show that individual measures of risk tolerance and expectations predict the share of stocks respondents invest in a hypothetical financial portfolio. Hoffmann, Post and Pennings (2013) and Merkle and Weber (2011) analyze the role of expectations and risk tolerance in trading behavior of individual investors rather than the share of stocks in household portfolios.

Second, the VRI survey includes batteries of questions that we purposely designed to produce estimates of preference and belief parameters that should help to explain the cross-sectional distribution of portfolio choices. These survey questions yield *quantitative* estimates of individual-level moments of subjective returns distribution and of individual-level values of preference parameters. These estimates can then be related to portfolio decisions in ways that are quantitatively interpretable relative to benchmark economic models.

Third, the design of the VRI allows careful consideration of response errors along a variety of dimensions. These include errors in measuring stock shares in both survey and administrative data and errors in eliciting preferences and expectations from survey responses.

These features—a large, broadly representative sample of stockholders together with quantitative measurements of the potential sources of heterogeneity in stockholding—make the VRI a unique platform for understanding why different households make different portfolio choices.

Section I describes the VRI sample and the measurements of assets and stock share. It Section II describes how we measure preferences and beliefs. To get individual-specific estimates of preference parameters, we use a modification of the Barsky, Juster, Kimball, and Shapiro (1997) approach of eliciting risk tolerance from hypothetical gambles over permanent income. To get individual-specific estimates of the moments of the perceived distribution of returns, we use both the Manski (2004) approach of eliciting points in the CDF of perceived returns together with individuals' estimates of expected returns.<sup>3</sup>

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<sup>3</sup> Survey measures of preferences have considerable external validity (i.e., that preference parameters explain a wide range of behaviors) and internal validity (i.e., test-retest validation and consistence across different measures). See Barsky, Juster, Kimball, and Shapiro (1997), Kimball, Sahm, and Shapiro (2008), Dohmen, Falk, Huffman and Sunde (2010), Dohmen, Falk,

Like many survey measures, preference and expectations are subject to response error. This paper uses a unified procedure accounting for response error to produce unbiased estimates of the subjective variables for both preferences and beliefs. Section III combines these estimates to explain the cross-section of stock shares. We find that the stock share is positively related to the individuals' perceived expected stock returns, is negatively related to their perceived standard deviation of the returns, and is positively related to their risk tolerance. These relationships are economically and statistically significant, they are robust across many specifications, and they are substantially less attenuated than corresponding estimates that do not take care of measurement error in the survey answers.

The paper uses two benchmark models of portfolio choice to assess the relationship between *quantitative* measures of preferences and beliefs—the classic Merton model and a richer lifecycle model that makes more realistic assumptions about the environment for portfolio choice. The relative magnitude of the estimated importance of expected returns, standard deviation of returns, and risk tolerance for explaining portfolio shares is quite close to the predictions of benchmark theory, though the absolute magnitudes are much smaller than theory would predict. We call the finding that portfolio shares have a damped response to preferences and beliefs the “attenuation puzzle.” We find that there is a significant cross-sectional heterogeneity in the size of the attenuation. In particular, the attenuation is substantially less

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Huffman, Sunde, Schupp, and Wagner (2011), and Josef, Richter, Samanez-Larkin, Wagner, Hertwig, and Mata (2016) for evidence both of external and internal validity. Recent evidence suggests survey measures of risk preferences show more stability than measures based on small-stakes lottery experiments (Lönnqvist, Verkasalo, Walkowitz and Wichardt, 2015). Similarly, probabilistic measures of expectations have predictive validity (Hurd, 2009; Kezdi and Willis, 2009).

severe for respondents with an MBA degree. To investigate underlying reasons for attenuation, we use features of the VRI to examine stability of expectations and awareness of portfolios.

Hence, though the results show that it is possible to use survey responses about economically-relevant subjective variables to explain meaningful features of stock holding, the actual distribution of stockholding varies less with the subjective variables than theory would predict given the measured heterogeneity in subjective variables. The paper's contributions to the measurement of preferences and beliefs address head-on potential explanations for the attenuation puzzle based on the measurement and modeling of preferences and beliefs:

- First, in contrast to measures of risk tolerance and expected returns based on loose or vague attitudinal scales, this paper presents quantitative estimates of the preference and belief parameters that theory mandates should determine portfolio choice.
- Second, this paper uses a statistical approach where the estimated individual-specific preference and belief parameters are by construction uncorrelated with the measurement errors that arise from the response errors that are inherent in eliciting subjective responses from individuals.

Hence, the estimated relationships presented in the paper are not subject to the attenuation bias that arise from having regressors that are only loose proxies for the variables of interest or that are subject to classical errors in variables. Consequently, our findings imply that the attenuation puzzle is a feature of investor behavior that is not well-captured by benchmark models.

## I. VRI Data and Stock Share Measurement

### A. VRI sample and wealth measurement

The Vanguard Research Initiative (VRI) consists of linked survey and administrative data of account holders who have non-negligible financial assets at Vanguard, are at least 55 years old, and use the Internet to access their Vanguard accounts. This last requirement is necessary because the VRI is an Internet survey. The VRI is an individual level survey, but it includes questions about household-level wealth and income as well as questions about spouses' or partners' demographics and labor supply. The survey oversampled older account holders and singles. The VRI draws respondents from two lines of business—individual account holders and employer-sponsored account holders.<sup>4</sup> (See Appendix B for more details on the VRI surveys and sample.)

Features of the VRI that make it well-suited for this analysis. First, it has a new approach to wealth and portfolio measurement. Second, it provides a larger sample of respondents with relevant levels of assets and stock holding compared to leading surveys such as the Health and Retirement Study (HRS) and the Survey of Consumer Finances (SCF). Third, demographics of the VRI are nonetheless comparable to those with similar asset levels in the HRS and SCF.

The VRI survey measure of wealth is based on a comprehensive account-by-account approach. The survey first asked about types of accounts respondents have (e.g. IRA, checking, money market funds) and the number each type of account held by the respondent or her spouse. For each account they indicated owning, the respondents were asked to provide the balance as

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<sup>4</sup> The employer-sponsored are enrolled at Vanguard through 401(k) or similar defined-contribution accounts. While both individual and employer-sponsored account holders are selected via ownership of a Vanguard account, the selection into individual and employer-sponsored accounts is presumably quite different. We will present separate estimates to get a sense of whether selection matters for results.



well as the share of stock-market assets. When finished with all accounts, respondents were presented a summary table consolidating their responses and were invited to make corrections, if any. Measuring wealth and stock shares account by account matches the way respondents keep track of their own wealth, and it does not require them to sum balances across accounts to provide total figures for asset categories that are familiar to economists but less so to survey respondents. In contrast, the HRS and SCF—other leading surveys with state of the art wealth measurement—use account-by-account approaches but only for selected sets of account types. Item non-response in the wealth section of the VRI affects less than 1 percent of the observations.<sup>5</sup>

Table 1 compares the VRI sample to the HRS and SCF.<sup>6</sup> The HRS and SCF are nationally representative samples (of those above age 50 in the case of the HRS). Table 1 compares the VRI sample to the subsample of the HRS and SCF after imposing restrictions similar to VRI eligibility: being at least 55 years old, having access to Internet at home, and having at least \$10,000 financial wealth. The table shows the number of households, the number of stock holding households, average financial wealth and total wealth, average stock shares, and some demographic characteristics of the individuals responding each survey.

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<sup>5</sup> Summary statistics of the wealth measures are shown in Table A1 in the Appendix. Table A2 in the Appendix shows the summary statistics of the variables we use as controls in our analysis, together with the definition of those variables.

<sup>6</sup> This comparison with the HRS and the SCF draws on Ameriks, Caplin, Lee, Shapiro and Tonetti (2014a). See this for further details about the VRI sample and wealth measurement.

Table 1. Summary Statistics: VRI, HRS, and SCF

	VRI		HRS	SCF
	Entire sample	Analysis sample	VRI-eligible subsample	
<i>Household-level variables</i>				
Number of households	8,950	4,414	3,684	1,275
Number of stockholding households	8,636	4,323	2,356	1,216
Average financial wealth (\$'000)	1,207	1,148	578	970
Average total wealth (\$'000)	1,589	1,551	804	1,764
Average stock share among stockholders	0.56	0.56	0.55	0.46
<i>Respondent-level variables</i>				
Married	0.67	0.68	0.70	0.71
Male	0.64	0.65	0.56	0.79
Age	67.8	67.8	64.9	64.1
Less than college degree	0.30	0.26	0.51	0.45
College degree but not more	0.32	0.33	0.23	0.27
Post-college degree	0.38	0.41	0.26	0.28
Retired	0.56	0.60	0.53	0.34

Notes. For the HRS and SCF, the VRI-eligible subsamples are those who are not younger than 55, have access to the Internet at home, and have at least \$10,000 in non-transactional accounts. Respondent-level variables for the HRS refer to the financial respondents; for the SCF they refer to the household heads. Variables in the VRI measured in 2013; HRS and SCF are from 2012 and 2013, respectively. Respondent-level variables are {0,1} binary variables except for age.

The number of respondents in Survey 1 is substantially larger than the VRI-eligible subsample of the HRS and the SCF. The difference in the number of respondent in stock holder households is even larger: the comparable samples have slightly over 1,000 stock-holding households in the SCF and slightly over 2,000 in the HRS; the entire VRI sample has more than 8,000 stock holders and the sample used in our analysis has more than 4,000.

The demographic composition of the VRI sample is very similar to the parallel subsamples of the HRS and the SCF. Average total wealth and average financial wealth in the VRI are remarkably close to corresponding estimates from the SCF; the HRS estimates are lower. The average stock share in financial wealth among stock holders is very similar in the VRI and the HRS; the SCF estimates are somewhat smaller. VRI respondents are slightly less

likely to be married, and they are somewhat older, more educated and more likely to be retired. The differences in marital status, age and retirement are largely due to the fact that the VRI oversampled older individuals and singles. 65 percent of the VRI sample is male, compared to 79 percent in the SCF and 56 percent in the HRS. Within households, men are overrepresented as respondents: account holders in the VRI, financial respondents in the HRS, and household heads in the SCF.

### B. Measuring stock shares

Our analysis focuses on the share of stock market-based assets in total financial wealth.<sup>7</sup> The stock share in financial wealth is the weighted average of the stock shares of the accounts as reported by the respondents. Respondents who did not answer all of the account-by-account stock share questions were asked the overall stock share of their financial portfolio. Ninety-five percent of respondents answered all the account-by-account stock share questions; the distribution of stock share is very similar across the two groups.

Besides the overall stock share we also analyze the stock share in wealth held at Vanguard based on administrative data. The monthly history of accounts in the VRI administrative data breaks down the balance of each account into stock, bond and money market holdings. This break-down is not readily available for all accounts, so we imputed stock share when needed using information on the type of fund the account is invested in (e.g., for an account invested in a balanced fund, we assume 60% of stock share). The administrative stock

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<sup>7</sup> Specifying stock share in financial wealth is standard in the literature. Alternative measures may include housing wealth and human capital wealth in the denominator. We include such wealth items as control variables in the analysis and show that their inclusion leads to very similar results for the parameters of interest. We also show that our main findings are robust to including housing wealth as either risky or safe assets in the risky asset share calculation.

share measure is available both at around the time when the stock expectation questions are asked and also at the time when the survey measure of household portfolio is obtained (the wealth survey took place in the fall of 2013, while expectations were asked in the summer of 2014). At the same time, the administrative stock share measure corresponds to the subset of financial wealth held at Vanguard.

Figure 1A compares the stock share measures based on the survey and the administrative data that will be the main dependent variables for our analysis. The horizontal axis shows the administrative measure of the stock share in assets held at Vanguard at the time we measured expectations in Survey 3, while the vertical axis shows the Survey 1 measure of stock share in total financial assets.<sup>8</sup> The size of the marks on the figure is proportional to the Vanguard financial wealth of the respondents. The figure shows that many observations are near the 45-degree line, so as a practical matter either measure may provide similar inferences for many respondents. At the same time, the two are often different. Indeed, the correlation coefficient is only 0.61.<sup>9</sup> The two measures can be different for three main reasons. First, they are measured at different times, in summer 2014 versus fall 2013. Second, they refer to different sets of assets: Vanguard assets versus all financial assets. Third, they are measured in different ways: using administrative records versus answers to survey questions.

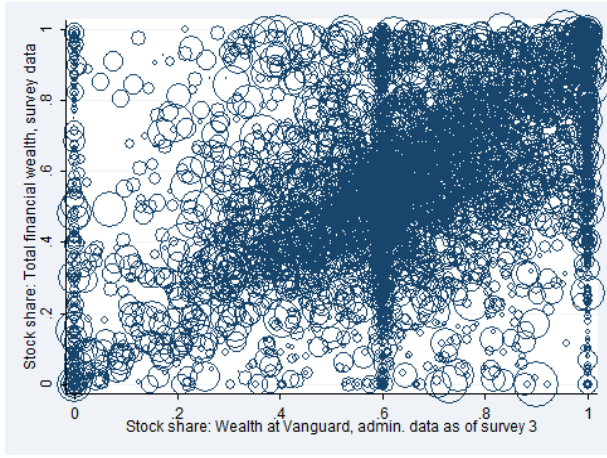
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<sup>8</sup> Due to the imputation used for the balanced funds, there is heaping at 60 percent in the administrative measure.

<sup>9</sup> All figures and correlation coefficients are weighted by Vanguard wealth. The unweighted correlation is 0.40, indicating that deviations are somewhat larger if wealth is low.

Figure 1. Stock Shares, Alternative Measurements Compared

A. Total Survey versus Vanguard Administrative



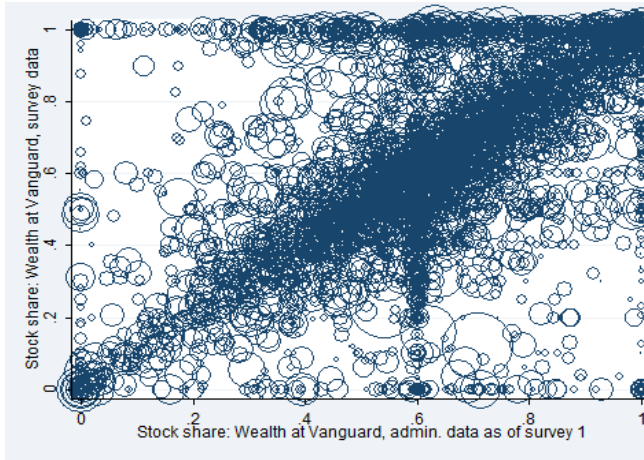
B. Different Time Periods, Vanguard Administrative



C. Total versus Vanguard Survey



D. Vanguard Survey versus Vanguard Administrative



Note: The variables on the horizontal and vertical axes are:

Panel A: the administrative measure of stock share at Vanguard at the time of Survey 3 versus the survey measure of stock share overall at the time of Survey 1 (the two main dependent variables in the analysis),

Panel B: the administrative measure of stock share at Vanguard at the time of Survey 1 versus the administrative measure of stock share at Vanguard at the time of Survey 3,

Panel C: the survey measure of stock share at Vanguard versus the survey measure of stock share overall,

Panel D: the administrative measure of stock share at Vanguard at the time of Survey 1 versus the survey measure of stock share at Vanguard.

The size of the marks on the figures is proportional to the Vanguard financial wealth of the respondents.

It is important to understand which of these differences matter. Figures 1B through 1D compare alternative stock share measures broken down by the time of measurement, the accounts covered, and the method of measurement (survey or administrative). First, consider the time

dimension. The horizontal axis of Figure 1B shows the stock share in assets held at Vanguard at the time of Survey 1 (fall 2013), while the vertical axis shows the same, measured at the time of Survey 3 (summer 2014). Almost all observations are on the 45 degree line, and the correlation is very strong (0.95), implying that the stock share changed little over this time period. As a practical matter, this means that changes in portfolios between administration of the VRI surveys is very small, so that differences in when the various questions were fielded is likely not that important. It is also of substantive interest that portfolio shares are so sticky, something that we see also in much longer intervals of the administrative data.

Second, consider the issue of whether or not the assets are held at Vanguard. Figure 1C plots stock share at Vanguard versus overall, both measured as survey responses in Survey 1. There is relatively high correlation in the stock shares (correlation = 0.81), so differences in portfolio shares across providers, though not trivial, is not the main source of the dispersion shown in Figure 1A. The difference can be small because account holders have most of their assets at Vanguard, or that they have asset compositions that are similar across Vanguard and non-Vanguard providers.

Third, Figure 1D shows the stock share at Vanguard at the time of Survey 1. Administrative data are on the horizontal axis and survey data are on the vertical axis. The dispersion is very much as in Figure 1A (correlation = 0.64). Some of the discrepancy is due to the fact that the survey asks about accounts held by the respondent and her or his spouse. When we restrict the sample to singles the correlation of the survey and admin measures is similar, at 0.66. Hence, it turns out that the main source of the dispersion is the deviation between survey and administrative measurements, not difference in timing or difference arising from Vanguard versus overall portfolios.

Several findings of independent interest emerge. First, based on the administrative data, portfolio shares are quite sticky over time. Second, the deviations of survey and administrative measures of portfolios suggest that individuals perceive different stock exposure than they have at any moment. Both these findings present challenges to standard theories of portfolio choice and therefore affect the interpretation of results relating portfolio choices to preferences and beliefs. We return to these issues after presenting the results.

## **II. Measuring Preferences and Expectations**

### **A. Measuring risk tolerance**

Survey 2 of the VRI included Strategic Survey Questions (SSQs) that ask respondents to make choices between hypothetical financial products under hypothetical situations. By specifying hypothetical situations that are independent from their own economic, health and family conditions, these SSQs enable us to better estimate structural preference parameters. This approach to measurement was pioneered by Barsky, Juster, Kimball, and Shapiro (1997) for measuring risk preference in the HRS and Ameriks, Caplin, Laufer, and Van Nieuwerburgh (2011) for measuring preferences surrounding long-term care. The approach is refined and extended in the VRI (see Ameriks, Briggs, Caplin, Shapiro and Tonetti, 2016, 2017). In this paper, we use the VRI's risk tolerance questions that pose gambles over consumption. The VRI risk preference questions ask about preference between having a certain level of consumption and a 50-50 chance of having double that level of consumption versus having it fall by  $x\%$ . It

then alters the downside risk  $x$  to partition respondents into risk tolerance groups.<sup>10</sup> Table A3 in Appendix A gives the exact wording of the risk tolerance question in the VRI.

The question is asked for two different levels of riskless consumption, \$100K and \$50K per year, and downside risks of 1/10, 1/5, 1/3, 1/2, and 3/4. Table 2 shows the distribution of the answers to the two questions. Most respondents have low tolerance for risk. About half of the respondents chose the first two categories, indicating that they would not accept a risk of more than 20% drop in their consumption to take a chance to double their consumption. Only a small fraction chose the last two categories with a risk of more than a 50% drop. Overall, the distribution is similar to the distribution of the answers to a similar question in the HRS except that the fraction of respondents in the two extreme categories (0-10% and 75-100%) is slightly lower in the VRI (see Kimball, Sahm, and Shapiro 2008 for HRS).

The table also shows that more respondents fall into the lower risk categories when riskless consumption is \$50,000 instead of \$100,000. We will handle this increase relative risk tolerance by positing a utility function with a subsistence level of consumption. Following Barsky, Juster, Kimball, and Shapiro (1997) and Kimball, Sahm and Shapiro (2008), we will use the multiple responses to identify the heterogeneity of the preference parameter and survey response errors (see details below).

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<sup>10</sup> The original HRS question has the same structure, but asks about gambles over life-time income rather than consumption.



Table 2. Risk Tolerance: Distribution of Responses to SSQ

Response category	Downside risk		Percent of answers	
	accepted	rejected	riskless consumption \$100K	riskless consumption \$50K
1	none	1/10	23	28
2	1/10	1/5	26	34
3	1/5	1/3	26	26
4	1/3	1/2	13	9
5	1/2	3/4	10	3
6	3/4	none	2	1
Total			100	100

Choice between two plans. Plan A guarantees \$W consumption next year. Plan B: doubles \$W with 50% chance and cuts it by a fraction x with 50% chance. \$W=100K or 50K, shown in the two columns; the x values are shown in second and third columns. 4414 observations.

Estimation of a cardinal risk tolerance parameter requires specifying a utility function.

We assume that the flow utility function is a generalization of CRRA with a subsistence level of consumption

$$u_i(c) = \frac{(c + \kappa)^{1-1/\theta_i}}{1-1/\theta_i}, \quad (1)$$

where subscript  $i$  denotes heterogeneity across individuals,  $c$  is consumption, the negative of  $\kappa$  is the subsistence level of consumption, assumed to be the same for all individuals, and  $\theta$  is the risk tolerance parameter.

For this utility function, relative risk tolerance ( $RRT_i$ ) is

$$RRT_i = \theta_i \frac{c + \kappa}{c} < \theta_i,$$

where the risk tolerance parameter  $\theta_i$  is relative risk tolerance in the  $\kappa = 0$  case. Empirically, the coefficient of risk tolerance is very close to what is implied by  $\theta_i$ , as the level of average wealth (Table 1) and annual income before retirement (\$90,000) are substantially larger than our estimate of  $-\kappa$ . At levels of consumption implied by the average before-retirement income, the

difference is less than 20%, and its variation between individuals is small. See Appendix Figure A1 for the relationship of relative risk tolerance and  $\theta$  as a function of consumption.

To parameterize the heterogeneity of the risk tolerance parameter, we assume that the parameter is distributed lognormally in the population according to

$$\log(\theta_i) = \bar{\theta} + u_{\theta_i}, \quad u_{\theta_i} \sim N(0, \omega_{u\theta}^2). \quad (2)$$

We model the measurement error as a log additive term to the parameter, such that

$$\begin{aligned} \log(\tilde{\theta}_{ij}) &= \log(\theta_i) + \varepsilon_{\theta_{ij}} \quad \text{for } j = 1, 2 \\ \varepsilon_{\theta_{ij}} &\sim N(0, \omega_{\varepsilon\theta_j}^2) \end{aligned} \quad (3)$$

where  $\theta_i$  is the true risk tolerance parameter for individual  $i$ ,  $\varepsilon_{\theta_{ij}}$  is measurement error, and  $\tilde{\theta}_i$  is the risk tolerance parameter that provides the basis for individual  $i$ 's response to the  $j^{\text{th}}$  question. Thus, in answering question  $j$  given the level of resource  $c$  and risk  $x$  that are associated with the risky gamble, the respondent compares

$$\frac{(c + \kappa)^{1-1/\tilde{\theta}_{ij}}}{1-1/\tilde{\theta}_{ij}} \quad \text{vs.} \quad 0.5 \frac{(2c + \kappa)^{1-1/\tilde{\theta}_{ij}}}{1-1/\tilde{\theta}_{ij}} + 0.5 \frac{((1-x)c + \kappa)^{1-1/\tilde{\theta}_{ij}}}{1-1/\tilde{\theta}_{ij}} \quad (4)$$

to determine whether to accept the risky gamble or not. This approach follows Kimball, Sahn, and Shapiro (2008). We carried out the estimation procedure jointly for risk tolerance and stock market expectations, so will defer discussion of estimation until Section II.C below.

## B. Measuring beliefs about stock returns

Survey 3 of the VRI asked about beliefs about the one-year return of the U.S. stock market, represented by a stock market index such as the Dow Jones Industrial Average (DJIA).

Respondents had to answer three questions: the expected return on the stock market in the 12 months following the interview ( $m$ ); the percent chance that the stock market will be higher in 12

months following the interview ( $p0$ ) and the percent chance that it will be at least 20% higher ( $p20$ ). The exact wording of the questions is in Table A4 in the Appendix.<sup>11</sup>

Answers to the expected value questions were constrained to be integers. Answers to the percent chance questions were constrained to be 5 point increments between 0 and 15 and between 85 and 100, and they were constrained to be 10 point increments between 15 and 85 (the set  $\{0,5,10,15,25,35,45,55,65,75,85,90,95,100\}$ ). Answers to percent chance questions tend to be rounded to the nearest ten when they are not constrained, with an especially large fraction answering 50 percent (Hurd, 2009). The VRI survey instrument requires people to round to other values; in particular, they cannot give 50 percent probabilities. The survey also requires that that  $p20 \leq p0$ .<sup>12</sup> The survey imposes no constraints on  $m$  versus  $p0$  and  $p20$ .<sup>13</sup>

Table 3 shows the summary statistics of the answers to the questions about the distribution of stock market returns. The survey responses for expected returns ( $m$ ) are distributed around the historical average of 4 to 7 percent depending on sample period, and their dispersion is moderate.<sup>14</sup> In contrast, most answers to the probability questions are lower than the historical probabilities, and they have substantial heterogeneity. A non-negligible fraction of the respondents gave a positive number to the expected return question ( $m$ ) and a less than 50

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<sup>11</sup> Bruine de Bruin et al. (2011) and Armantier et al. (2013) examine the reliability of the percent chance questions for inflation as well as how they relate to questions about point expectations of inflation.

<sup>12</sup> Respondents whose initial answer to  $p20$  violated this constraint are reminded of the constraint by the survey software and asked for a new reply to either  $p0$  or  $p20$  (or both).

<sup>13</sup> A randomly selected half of the respondents received the  $m$  question first, followed by  $p0$  and  $p20$ , while the other half received  $p0$  and  $p20$  first, followed by  $m$ . The distribution of the responses is slightly different across the two sequences. Nevertheless, we find similar relationships between the belief measures and portfolio choice from the two sequence groups.

<sup>14</sup> Individuals may use different sample windows for inferring expected returns (see Malmendier and Nagel 2011). The table shows some different windows for realized returns. Average returns are quite variable owing to the well-known problem of estimating the expected return on the market.

percent chance answer to the probability of a positive return ( $p0$ ). Taken together these answer patterns are consistent with many individuals implicitly applying a positive threshold when they answer the  $p0$  question (by thinking that the stock market goes up only if it goes up by at least some positive amount).<sup>15</sup>

Table 3. Stock Market Returns: Survey Responses versus Historical Statistics

	Survey answers				Historical statistics			
	Mean	25 <sup>th</sup> pctile	Median	75 <sup>th</sup> pctile	1959- 2014	1995- 2014	1995- 2009	2010- 2014
$m$	0.06	0.04	0.06	0.10	0.04	0.07	0.05	0.13
$p0$	0.51	0.25	0.55	0.75	0.58	0.65	0.53	1.00
$p20$	0.15	0.05	0.15	0.25	0.18	0.25	0.27	0.20

Notes.  $m$  is expected one-year ahead returns of the stock market index DJIA;  $p0$  is the probability that the DJIA would be higher a year from the date of the interview;  $p20$  is the probability that it would be higher by at least 20%. Historical statistics computed from yearly relative returns of the Dow Jones Industrial Average (year on year changes divided by base year value, first days of July in each year), deflated using the PCE chain price index (available beginning in 1959). Historical average values shown for  $m$ ; the fraction of years when positive or greater than 0.2 are shown for  $p0$  and  $p20$ . 4414 observations.

In order to use our data more efficiently and in a way that is more informative from a theoretical point of view we map the three survey responses,  $m$ ,  $p0$ , and  $p20$  into a perceived returns distribution. The procedure closely parallels that for the risk tolerance questions: the survey responses are based on individual beliefs drawn from normal distribution plus survey response error. We assume that individual  $i$  believes that yearly returns follow a lognormal distribution with individual-specific mean and standard deviation of log stock returns of  $\mu_i$  and  $\sigma_i$ . Similar to how we handle the cross-sectional distribution of the risk tolerance parameter, these parameters are drawn across individuals as

<sup>15</sup> Glaser, Langer, Reynders and Weber (2007) document a similar pattern when they compare stock market expectations elicited in terms of returns versus prices. They label the phenomenon as “framing effect,” and our explanation can be viewed as a source of such a framing effect. Note that, although skewed returns could explain the phenomenon we observe, it is an unlikely explanation. The combination  $m_i > 0$  and  $p0_i < 0.5$  would correspond to long positive tails, implying mean above the median and infrequent large gains. This skewedness is the opposite of what one would expect from a “black swan” theory of infrequent stock market crashes.

$$\begin{aligned} \mu_i &= \bar{\mu} + u_{\mu i}, \\ \sigma_i &= \bar{\sigma} + u_{\sigma i}, \end{aligned} \quad \begin{pmatrix} u_{\mu i} \\ u_{\sigma i} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \omega_{u\mu}^2 & \rho_{\mu\sigma} \omega_{u\mu} \omega_{u\sigma} \\ \cdot & \omega_{u\sigma}^2 \end{pmatrix} \right). \quad (5)$$

Individuals answer the survey questions  $m$ ,  $p0$  and  $p20$  based on their beliefs, but their answers contain survey noise, that is, measurement error specific to the survey situation. Using the structure of the survey questions on expected returns and the two points of the probability distribution, applying the assumption of lognormal returns, and adding survey response error yields

$$\tilde{m}_i = \mu_i + \varepsilon_{mi}, \quad \varepsilon_{mi} \sim N(0, \omega_{\varepsilon m}^2) \quad (6)$$

$$\tilde{p}_{0i} = \Phi\left(\frac{\mu_i}{\sigma_i} + \varepsilon_{0i}\right), \quad \varepsilon_{0i} \sim N(\psi, \omega_{\varepsilon p}^2) \quad (7)$$

$$\tilde{p}_{20i} = \Phi\left(\frac{\mu_i - 0.2}{\sigma_i} + \varepsilon_{20i}\right), \quad \varepsilon_{20i} \sim N(0, \omega_{\varepsilon p}^2) \quad (8)$$

where  $\tilde{m}_i$ ,  $\tilde{p}_{0i}$ , and  $\tilde{p}_{20i}$  are the error-ridden variables that determine survey responses. Survey error is assumed to be independent across the three answers, with mean zero except for  $p0$  where its mean is  $\psi$ , which allows for the documented differences between  $m$  and  $p0$ . An interpretation of  $\psi$  is that, on average, respondents answer the question about positive returns ( $p0$ ) as if they had some positive threshold in mind instead of zero ( $\Phi(-\psi / \sigma_i)$ ,  $\psi < 0$ ). The variables  $\tilde{m}_i$ ,  $\tilde{p}_{0i}$ , and  $\tilde{p}_{20i}$  are before rounding. Recall that the VRI probability scale is for rounded responses. Similarly, as discussed above, the risk tolerance questions yield discrete responses. The next section discusses how our estimation procedure handles this issue.

### C. Joint estimation of heterogeneity in stock market expectations and risk tolerance

Given the models of heterogeneity in preferences and beliefs (equations (2) and (5)), the structural interpretation of the survey questions together with the additive survey response errors ((3), (4), (6), (7) and (8)), we can now move to estimation of the model. The parameters to be estimated are  $\bar{\theta}, \bar{\mu}, \bar{\sigma}, \omega_{u\theta}^2, \omega_{u\mu}^2, \omega_{u\sigma}^2, \rho_{\mu\sigma}, \psi, \omega_{\varepsilon\theta 1}^2, \omega_{\varepsilon\theta 2}^2, \omega_{\varepsilon m}^2, \omega_{\varepsilon p}^2$ . We allow for  $\bar{\theta}, \bar{\mu}, \bar{\sigma}$ , and  $\psi$  to vary with covariates. Additionally, we allow the beliefs about returns to depend on risk preference, so the covariates of  $\bar{\mu}$  and  $\bar{\sigma}$  include the latent  $\theta_i$ . The estimation method is maximum likelihood. It allows for interval responses to the risk tolerance question and the returns questions. Appendix C shows the likelihood function.

Table 4. Distribution of Preferences and Beliefs

		Mean	Standard deviation	25 <sup>th</sup> pctile	Median	75 <sup>th</sup> pctile
<i>Preferences</i>						
Risk tolerance parameter	$\theta_i$	0.41	0.33	0.20	0.32	0.50
Subsistence consumption	$-\kappa$	17,000				
<i>Beliefs</i>						
Mean of return	$\mu_i$	0.06	0.06	0.02	0.06	0.10
Standard deviation of return	$\sigma_i$	0.12	0.03	0.10	0.12	0.14

Notes. Statistics are calculated from the estimated parameters in Table A5; see the notes to Table A5 for more detail.

Table 4 shows key estimated statistics of the distribution of preferences and beliefs based on the estimated statistical model of preferences, beliefs, and response error. Table A5 in the appendix shows the estimates of the underlying parameters of the model.<sup>16</sup> The subsistence level of consumption ( $-\kappa$ ) is estimated to be \$17,000. The negative value of  $\kappa$  generates decreasing

<sup>16</sup> The summary statistics in Table 4 are from estimates without covariates. Appendix Table A6 reports the estimates of the statistical model with covariates.

relative risk aversion as in the basic/luxury good model of Wachter and Yogo (2010).<sup>17</sup> The estimated mean of the risk tolerance parameter ( $\theta$ ) implies low risk tolerance on average. A respondent with the mean level of  $\theta$  and  $\kappa$  has relative risk tolerance 0.34 (relative risk aversion 2.9) when the consumption level is \$100,000. In terms of the SSQ question, she would be indifferent between a fixed consumption of \$100,000 and the 50-50 gamble of doubling that consumption and losing 20 percent. There is a considerable heterogeneity in risk tolerance. At the 25th percentile of risk tolerance parameter, the point of indifference is the downside risk of losing 13 percent; at the 75th percentile the point of indifference is the downside risk of losing 29 percent. These numbers indicate higher levels of risk tolerance than in a representative sample of Americans older than 50 years of age. Kimball, Sahm and Shapiro (2008) estimate the corresponding risk tolerance percentiles (25th, 50th and 75th) to imply indifference to 7, 12 and 20 percent of downside risk, respectively.

Beliefs about mean stock returns are in line with historical mean returns, on average. Beliefs about standard deviation are slightly lower than the historical value of 0.16. Heterogeneity in perceived mean returns ( $\mu$ ) is substantial, with the lowest 25 percent believing expected returns to be 2 percent or less and the top 25 percent believing 10 percent or more. At the same time, estimated heterogeneity in the perceived standard deviation of stock returns ( $\sigma$ ) is small, perhaps because it is easier for people to estimate the second moment of the returns distribution than the first moment, as pointed out by Merton (1980).<sup>18,19</sup>

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<sup>17</sup> The design of the SSQ does not allow heterogeneity in  $\kappa$  to be readily identified, although it tightly identifies its mean.

<sup>18</sup> According to our estimates heterogeneity in preferences and beliefs are weakly related. More risk tolerant respondents believe that stock returns are slightly higher, but we don't find association of risk tolerance and beliefs about the standard deviation of returns. Beliefs about the mean and the standard deviation of returns are weakly positively correlated.

Based on the estimated distribution summarized in Table 4, 17 percent of the population expects negative stock returns. As we will see, this part of the population holds less stock than on average, but still has substantial stock market exposure. Symmetrically, 17 percent expect returns to be larger than 12 percent, rates of return that should make people hold the vast majority of their wealth in stocks given the distribution of risk and risk preferences. Though this part of the population holds more stock than on average, very high stock shares are uncommon. Taken together, these facts suggest that expectations translate into stock shares in an attenuated fashion, a finding that our analysis will verify in the next section.

The Table 4 results take into account substantial estimated survey noise. Again, the parameters of the survey noise distributions are presented in Appendix Table A5. To understand the magnitude of noise, consider the differences in terms of the survey responses of individuals with the estimated averages of latent preferences and beliefs, one without measurement error and one with a positive standard deviation unit shock of measurement error. A one standard deviation unit measurement error in the first risk tolerance SSQ would make the survey response imply a point of indifference of a 38% drop of consumption instead of the 20% implied by an error-free answer. A one standard deviation unit measurement error in the second risk tolerance SSQ would make the response imply an indifference point of 27% instead of 17%. One standard deviation unit measurement error in the response to the expected stock returns question would result in a

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<sup>19</sup> Preferences and beliefs are significantly related to observable right hand side variables in our sample (Table A5 in the Appendix). However, when interpreting these associations, one has to keep in mind that the VRI sample is selected on wealth and stock ownership. For example, sample selection may explain the negative correlation of wealth and stock market expectations. Almost all households in the VRI sample have nonzero stockholding. With fixed costs of stock market participation wealth should matter at the extensive margin on top of expectations. As a result, we expect wealthier stockholders to have lower expected returns than less wealthy stockholders.



response of 14% instead of 6%; one standard deviation unit measurement error in the stock market probability answers would change  $p0$  responses to 67% from 48% and  $p20$  responses to 25% from 12%. The estimated bias of the measurement error in the  $p0$  response ( $\psi$ ) suggests that, on average, people think of positive gains only when they exceed 4 percent when answering the  $p0$  question.<sup>20</sup>

#### D. Estimating individual-specific cardinal proxies of risk tolerance and beliefs

In the previous sections, we show how to separately identify the true heterogeneity in preferences and beliefs and the survey response errors in the survey measures of them. In this subsection, we explain how we construct the individual-specific belief and preference parameters based on those estimates that are immune from the standard effects of using generated regressors.

##### 1. *Constructing individual-specific preference and belief parameters*

Using the estimation results we calculate individual-specific proxy variables  $\hat{\mu}_i$ ,  $\hat{\sigma}_i$  and  $\hat{\theta}_i$ .

These proxies are the expected values of the corresponding latent variables: the individual-specific expected value and standard deviation of the distribution of stock market returns perceived by the individual  $(\mu_i, \sigma_i)$ , and the individual-specific latent risk tolerance parameter ( $\theta_i$ ). They are expected values conditional on the individual's responses to the survey questions on stock market returns  $(m_i, p_{0i}, p_{20i})$  and to the SSQ's with the two hypothetical gambles. To get these expected value of the latent individual-specific parameters conditional on the survey

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<sup>20</sup> Allowing for covariates (Appendix Table A6),  $\psi$  is estimated to be substantially less negative among more educated and wealthier people, indicating that their threshold value is closer to the nominal threshold zero.

response and the statistical model, there are two steps. First, the distribution of the latent variables conditional on the observed responses can be obtained from the likelihood function using Bayes' theorem. Second, integrating out this function yields the individual-specific proxy variables ( $\hat{\mu}_i$ ,  $\hat{\sigma}_i$  and  $\hat{\theta}_i$ ) as the conditional expectations of the latent variables given the observed survey responses. These proxy variables deal with measurement error in survey responses. Appendix C spells out these steps in detail.

## *2. Using individual-specific preference and belief parameters in regressions*

Our aim is to use the survey-based estimates of individual-specific parameters to explain heterogeneity in portfolio choice. Note that this proxy error is not conventional errors in variables. Because each proxy is a conditional expectation, which is basically a projection, this measurement error is uncorrelated with the proxy (and correlated with the true latent variable). Hence, it can be included on the right-hand side of a regression without creating an attenuation bias (see Kimball, Sahm and Shapiro, 2008).

When the regressions include other covariates as well the OLS estimates are unbiased if the proxies are estimated conditional on those covariates, too. We therefore estimate two sets of proxies. The first set is conditional on the survey answers to the risk tolerance and the stock market belief questions only. The second set is conditional on other covariates as well. We use the second set of proxy estimates as right-hand-side variables in regressions that also include those covariates. In the next section, we present such regressions to explain portfolio behavior based on our estimates of preferences and beliefs.

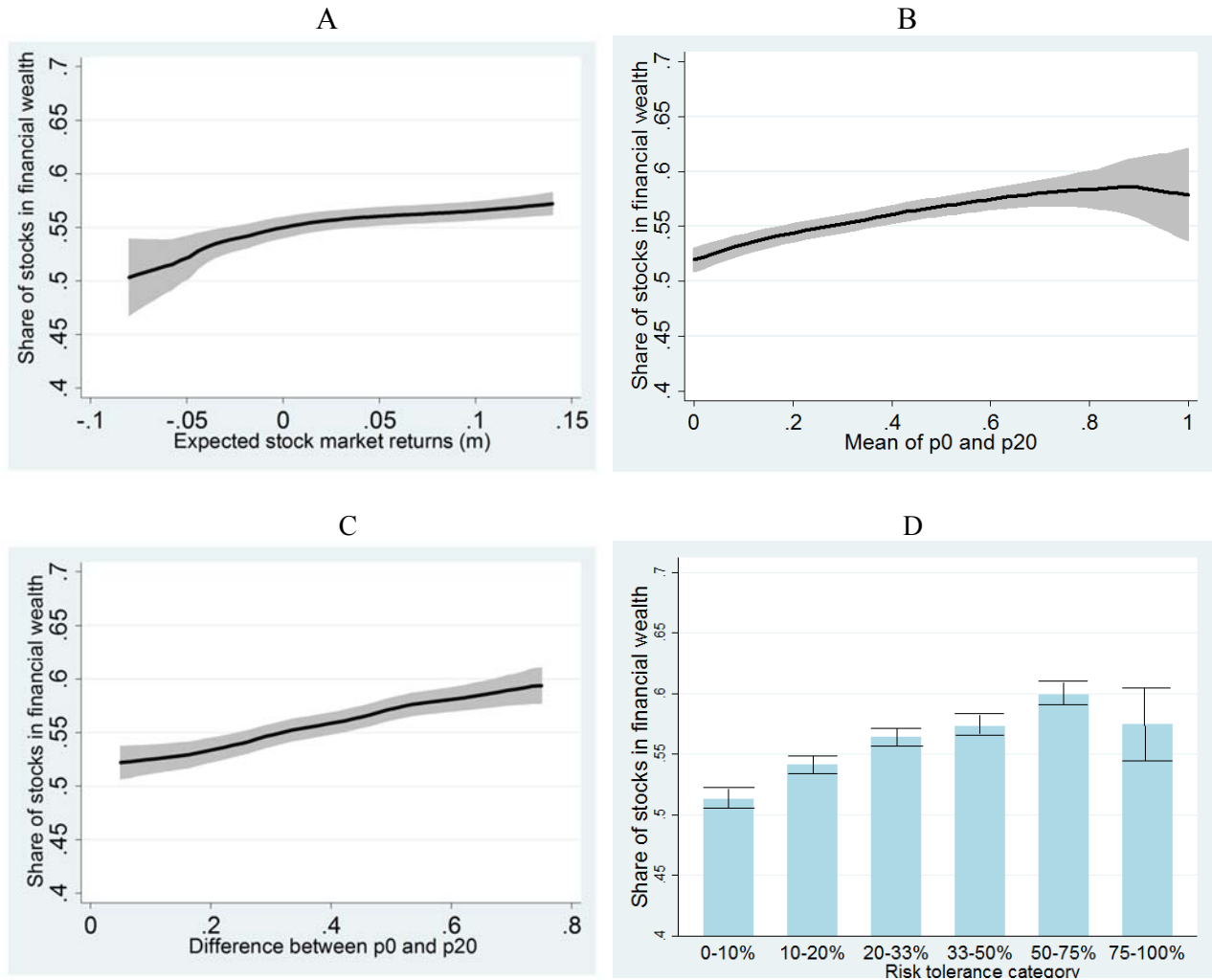
### III. Explaining Heterogeneity in Portfolio Choice

#### A. Stock share and answers to survey questions

Before turning to the regressions based on our structural estimates of the latent preferences and beliefs, we investigate the relationship between the stock share of household portfolios and the raw survey responses. Figure 2 shows non-parametric regressions of the stock share in total financial assets on the survey answers to expected stock market returns ( $m_i$ ), the average between the probability that the stock market would go up and that of an increase of 20 percent or more  $((p_{0_i} + p_{20_i})/2)$ , the difference between those two  $(p_{0_i} - p_{20_i})$ , and the answer to the risk tolerance question with income level \$100,000. (Figure A2 shows the analogous nonparametric regression results on  $p_{0_i}$  and  $p_{20_i}$  separately.)

The results indicate a positive relationship between the stock share of household portfolios and expected stock market returns ( $m_i$ ) and the mean of the two probability responses  $((p_{0_i} + p_{20_i})/2)$ . The stock share is also positively related to the difference between the responses to the probability questions  $(p_{0_i} - p_{20_i})$ , suggesting a negative relationship with perceived risk of stock returns. Finally, the stock share is monotonically positively related to the answers to the risk tolerance question except for the last categories that has relatively few responses, suggesting a monotonic positive relationship with risk tolerance. Hence, the relationship between the raw survey responses and the stock share has the direction benchmark theories of portfolio choice would suggest.

Figure 2. Stock Share versus Raw Survey Responses



We also estimate linear regressions with survey measure and the administrative measure of stock share as alternative left-hand-side variables and the same right-hand-side variables entered with and without the control variables that include detailed measures of demographics, education, employment, income, wealth, as well as background risks of long-term care and longevity. The results are included in Tables A7 and A8 in the Appendix. The results imply statistically significant relationships of stock share with the survey answers with or without the control variables. The magnitudes of the associations are difficult to interpret because not all

measures have a cardinal interpretation and because of the presence of survey noise. These problems are addressed in the next section.

### B. Stock share and cardinal proxies of expectations and risk tolerance

Our more structural analysis has two goals. First, it relates the stock share of household portfolios to cross-sectional heterogeneity in preferences and expectations in a way that is related to portfolio choice theory thus making magnitudes easier to interpret. Second, it aims at incorporating survey noise in the estimation thus reducing its effect on the estimated magnitudes. This is a structural analysis in the sense that it makes use of additional assumptions in order to relate stock shares to heterogeneity in latent preferences and expectations. The analysis is still reduced form in the sense that it aims at uncovering associations without claims for causality. Nonetheless, since the explanatory variables are proxies that have cardinal interpretations relevant for economic theories, they potentially convey much more information than the relationship of raw survey responses to economic outcomes.

Start from a general function of the solution of optimal stock share

$$s_i^* = s^*(\mu_i, \sigma_i, \theta_i, \kappa, x_i, u_i) \quad (9)$$

where  $\mu_i$  and  $\sigma_i$  are the beliefs of person  $i$  about the mean and the standard deviation of one-year-ahead stock returns,  $\theta_i$  is the risk tolerance parameter,  $x_i$  is a vector of wealth, demographic variables and other risk factors that are measured in our data, and  $u_i$  combines all unobservables. We assume that unobservables are independent of observables.

The relative deviation of  $s^*$  around its mean value is related to relative deviations of the other variables around their mean values, holding values of  $x_i$  constant by

$$\frac{s_i^* - \bar{s}^*}{s^*} \approx \beta_0 + \beta_1 \frac{\mu_i - \bar{\mu}}{\bar{\mu}} + \beta_2 \frac{\sigma_i - \bar{\sigma}}{\bar{\sigma}} + \beta_3 \frac{\theta_i - \bar{\theta}}{\bar{\theta}} + \beta_4 x_i + u_i. \quad (10)$$

The coefficients approximate the first derivatives of the function around the mean values, with  $\beta_1 = \partial \tilde{s}^* / \partial \tilde{\mu}$ ,  $\beta_2 = \partial \tilde{s}^* / \partial \tilde{\sigma}$  and  $\beta_3 = \partial \tilde{s}^* / \partial \tilde{\theta}$ , where the tilde denote relative differences from mean values. This approximation is a way of log-linearizing the function that allows observations with nonpositive values of some of the variables, which is relevant for  $\mu_i$  in our case. We linearize about the risk tolerance parameter rather than relative risk tolerance to avoid the ambiguity that relative risk tolerance depends on the level of consumption.

We estimated (10) using the observed stock share  $s_i$  to approximate the target stock share  $s_i^*$  and the individual proxies  $\hat{\mu}_i$ ,  $\hat{\sigma}_i$  and  $\hat{\theta}_i$  approximating the latent variables  $\mu_i$ ,  $\sigma_i$  and  $\theta_i$  as described earlier. We estimated the equation by OLS both with and without covariates.<sup>21</sup> When we controlled for covariates in the stock share equation we entered the structural parameters that were estimated conditional on the same covariates. Kimball, Sahm, and Shapiro (2008) show that it is necessary to construct the proxies conditional on the same covariates as included in the main regression to deliver unbiased coefficient estimates. As the proxies are generated regressors, we estimated the standard errors by bootstrapping the entire estimation procedure including the structural estimation of the model underlying the proxies. We estimated two versions of each regression: one with the survey measure of the share of stocks in total financial wealth on the left hand side and one with the administrative measure of stock share in wealth held at Vanguard. The main results are in Table 5. Table A9 in the Appendix shows the detailed results.

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<sup>21</sup> We do not use a Tobit-type procedure to account for the truncation at 0 and 1 because there are very few observations (less than 2 percent of the sample) at these boundaries.

The estimates show that the share of stocks is positively related to the perceived mean of stock market returns, negatively related to the perceived standard deviation of stock market returns, and positively related to the risk tolerance parameter. The estimated coefficients are statistically significant with the survey measure of stock shares on the left hand-side, but they are smaller and less significant in the administrative stock share regressions. In both cases the coefficients are very similar whether we enter them with or without the covariates.<sup>22</sup>

According to the point estimates, a one percent higher perceived mean is associated with one twentieth of a percent higher stock share; a one percent higher perceived standard deviation is associated with around one tenth of a percent lower stock share; and a one percent higher risk tolerance parameter is associated with one thirtieth of a percent higher stock share. Converting the relative magnitudes to absolute ones, our estimates imply that for the stock share to be higher by 1 percentage point expected returns need to be higher by 2.1 percentage points, the perceived standard deviation needs to be lower by 2.4 percentage points, or the risk tolerance parameter needs to be higher by 0.24.<sup>23</sup>

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<sup>22</sup> Table A9 in the Appendix shows that most of the coefficients on the other covariates are in line with prior expectations: the stock share is smaller in the employer-sponsored subsample and larger for wealthier and more educated individuals, especially for those with an MBA. Some of the other parameters are insignificant: the coefficient on the probability of needing long-term care (a factor of background risk), for example, is negative but insignificant.

<sup>23</sup> Comparing our estimates to the literature is not straightforward as most papers do not have cardinal proxies for the expectations and risk tolerance variables, and those that do estimate functional forms that are different from ours. Wherever we can make the comparison we find magnitudes that are very similar to our estimates. The closest to our specification are the estimates of Amromin and Sharpe (2012). On a sample of stockholders with positive expected returns they regress the log of the stock share on the log of their proxies of  $\mu$  and  $\sigma$ . Their point estimates are +0.04 and -0.11, respectively. These magnitudes are very close to ours. The results of the Tobit model of Vissing-Jorgensen (2003), estimated on a sample of investors, imply that one percentage point higher returns expectations are associated with about 0.5 percentage point higher equity share. Kezdi and Willis (2011) estimate a coefficient of 0.3 in a truncated regression model estimated on a representative sample with stock shares on the left hand-side.

Table 5. Stock Shares versus Cardinal Proxies for Preferences and Beliefs

	Survey Stock Share		Administrative Stock Share	
	(1)	(2)	(3)	(4)
$\hat{\mu}_i$	0.058*** (0.010)	0.055*** (0.009)	0.052*** (0.008)	0.047*** (0.008)
$\hat{\sigma}_i$	-0.093* (0.046)	-0.083 (0.051)	-0.068 (0.040)	-0.083* (0.038)
$\hat{\theta}_i$	0.034*** (0.009)	0.033** (0.010)	0.012 (0.010)	0.013 (0.010)
constant	-0.001 (0.008)	1.136 (0.649)	-0.001 (0.007)	0.803 (0.519)
covariates	N	Y	N	Y
R <sup>2</sup>	0.019	0.045	0.013	0.038
N	4414	4414	4414	4414

Notes. Stock share in total financial wealth (survey measure) and in Vanguard accounts (administrative measure) are regressed on proxies for the expected stock returns ( $\hat{\mu}_i$ ), perceived standard deviation of stock returns ( $\hat{\sigma}_i$ ), and the risk tolerance parameter ( $\hat{\theta}_i$ ). All variables are expressed as relative differences normalized to their mean values (as specified in equation (10)). Control variables: married, male, age, whether respondent comes from the employer-sponsored subsample, education (below college; college; MBA; PhD, other higher degree); log financial wealth, log wage, dummy for owning a house, log annuity income (Social Security and DB pensions) for retired, log expected annuity income for non-retired; dummy for retired, log home stock; subjective probability of needing long-term care, and longevity expectations. Bootstrap standard errors in parentheses. \*, \*\*, and \*\*\* implies significance at 5%, 1% and 0.1% level, respectively.

Table 6 shows results of analogous estimations that do not take care of measurement error in the survey answers (Table A10 in the Appendix contains all results). Instead of the cardinal proxies  $\hat{\mu}_i, \hat{\sigma}_i, \hat{\theta}_i$ , these regressions include the raw survey answers to the stock market expectation question ( $m_i$ ), a crude transformation of the probability answers to approximate

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Our log-linearized estimates imply that, around its mean, a one point difference in  $\mu$  is associated with a 0.45 percentage point difference in stock shares. In a Tobit model of stock shares that combines the extensive and intensive margins Kimball, Sahm and Shapiro (2008) find a small magnitude for the association with the cardinal proxy of risk tolerance.



perceived risk,<sup>24</sup> and the median value of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions ( $\kappa$  set to zero). The coefficient estimates are qualitatively similar to the baseline results reported in Table 5 above, but the magnitudes are considerably attenuated. The absolute values of the point estimates are one third to one half of the baseline estimates. These results are consistent with substantial measurement error in the raw survey answers. They show the importance of taking into account measurement error in the construction of the proxies and in using them in econometric models.

Table 6. Stock Shares Versus Error-Ridden Cardinal Measures of Preferences and Beliefs.

	Survey Stock Share		Administrative Stock Share	
	(1)	(2)	(3)	(4)
$m_i$	0.017*** (0.004)	0.020*** (0.004)	0.020*** (0.004)	0.021*** (0.004)
$\hat{\sigma}_i$	-0.029*** (0.006)	-0.019** (0.007)	-0.025*** (0.006)	-0.020** (0.006)
$\hat{\theta}_i$	0.021*** (0.005)	0.020*** (0.005)	0.013** (0.004)	0.013** (0.004)
constant	-0.001 (0.007)	1.120* (0.565)	-0.000 (0.006)	0.781 (0.507)
covariates	N	Y	N	Y
R <sup>2</sup>	0.013	0.039	0.012	0.038
N	4414	4414	4414	4414

Notes. This table reports estimates without taking care of measurement error in the cardinal proxies. Left-hand-side variables and covariates as in Table 5. Main right-hand-side variables are the raw survey answers to the stock market expectation question ( $m_i$ ), a crude transformation of the probability answers to approximate perceived risk ( $\hat{\sigma}_i = 0.2 / (\Phi^{-1}(p_{0i}) - \Phi^{-1}(p_{20i}))$ ), and the median value of the CRRA risk tolerance parameter that corresponds to the answers to the first set of the risk tolerance questions ( $\kappa$  set to zero). Bootstrap standard errors in parentheses. \*, \*\*, and \*\*\* implies significance at 5%, 1% and 0.1% level, respectively.

<sup>24</sup>  $\hat{\sigma}_i = 0.2 / (\Phi^{-1}(p_{0i}) - \Phi^{-1}(p_{20i}))$ . The denominator replaced with 0.2 if zero to obtain  $\hat{\sigma}_i = 1$ , which is larger than the maximum of all other values. This imputation affects less than 10% of the observations. Alternative imputations that replace the denominator with other values yield very similar estimates.

Table A11 shows estimates analogous to our benchmark model presented in Table 5 above for the employer-sponsored subsample. Self-selection to Vanguard is arguably substantially less severe in this subsample. However, the differences are small, suggesting that selection bias is unlikely to have a substantial effect on our main results.<sup>25</sup>

### C. Interpreting the magnitudes

How might one evaluate the estimates relative to an economic model? The simplest model of Merton (1969) with CRRA utility would imply that the coefficient on  $\log \mu$  should be 1, the coefficient on  $\log \sigma$  should be -2, and the coefficient on  $\log \theta$  should be 1 again. The same implications hold if we modify the utility function in the Merton model to incorporate the subsistence level of consumption as in equation (1) above, or if we include deterministic labor income (Merton, 1971).

The *relative* magnitudes of the estimated coefficients report in Table 5 are remarkably close to these theoretical implications of the Merton benchmark. In the regressions on the survey measure of stock share, the coefficient on the (approximately log-linearized) expected value and risk tolerance proxies are close to each other, and the coefficient on the standard deviation proxy is close to be negative two times their magnitudes. At the same time, the magnitudes are indeed smaller than in the benchmark model: each estimate is about one twentieth of what a simple theory implies.

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<sup>25</sup> Tables A12 through A15 in the Appendix show that results from analogous regressions are very similar in various subsamples, such as the sample of individuals that joined Vanguard with their private accounts, the sample of individuals with high share of household wealth held at Vanguard, and the sample of individuals with directly held stocks. Table A16 shows that the results are similar even if we include housing wealth either as a safe asset or as a risky asset in the calculation of the share of risky assets.

In principle, the attenuation bias may arise from classical errors in variables on the right hand-side or appropriate non-classical errors in the left hand-side variable. Recall that our measures of beliefs and preferences already take care of substantial survey noise that arise from noisy responses conditional on the latent variables. While it is of course possible for those latent variables to exhibit additional noise, due to, for example, mood effects, that noise would have to be extremely large for the observed attenuation. We believe that the magnitude of the attenuation and its similar strength across the coefficients call for an explanation beyond these measurement issues.

The Merton model is a simple and useful benchmark. However, it has assumptions that are very far from the way people invest in our sample as it requires continuous rebalancing, no background risk, and it allows for unlimited leverage and short sales. We therefore investigate whether adding these realistic features would move the predictions of the benchmark model more in line with what we observe in the data.

The model we use is a life cycle portfolio choice model in discrete time, with consumption and investment decisions at the yearly frequency and portfolio shares constrained to be between zero and one. The model incorporates background risk in income and longevity. Similarly to the Merton model, it has a risky asset (stocks) and a risk-free asset (bonds), and it is a model of demand, taking returns on those assets as given. It has no closed-form solution and is solved recursively. Our model can be viewed as a generalization of the model of Cocco, Gomes and Maenhout (2005). Appendix D contains its details and main results.

The lifecycle model of portfolio choice implies magnitudes that are similar to the Merton model in the range of interior solutions, and it implies corner solutions of zero and 100 percent stock shares otherwise. Our estimates of these associations are substantially weaker. Stock share

is close to fifty percent among those who expect non-positive stock returns in our sample, and it is substantially less than 100 percent on average among people with very high expectations even with low perceived risk and low risk tolerance. In the theoretical model, the share of stocks increases from zero to 100 percent if expected returns raise from the risk-free rate to seven percentage points above the risk-free rate even for investors that are in the top third of our estimated distribution of risk aversion and perceived risk. Our estimates imply that a seven percentage point difference in expected returns corresponds to less than a four percentage point difference in stock shares. Again, our estimate is about one twentieth of the magnitude implied by theory. Differences between the model and our estimates with respect to the role of perceived risk and risk tolerance are similarly large.

Risk preferences, expected stock returns, and the risk of stock returns are fundamental elements of any portfolio choice model. People should not hold stocks if they think their returns will be lower than returns on risk-free assets, and they should avoid or embrace risk in their investment decisions in a way that is in line with their choices in other gambles with money. These features are implied both by the very stylized Merton model and our more realistic life cycle model, yet our estimates do not deliver them. The two portfolio choice models imply similar magnitudes in the range of an interior solution without short sales and leverage, and our estimates fall short of those magnitudes.

#### D. Potential Explanations of the Attenuation Puzzle

The attenuated response of stock shares to beliefs and preferences is responsible for observing individuals who think that stocks are dominated in return often holding a high fraction of their

portfolio in stock. Symmetrically, this attenuated response makes many individuals who think that stocks have very high returns hold too little stocks. What might account for such behavior?

Several theories regarding belief formation or portfolio management proposed in the literature may be responsible for the attenuation. One possibility is the extrapolation bias, that households put a large weight on returns they experienced recently in forming their expectations (see Greenwood and Shleifer, 2014, for evidence from stock market expectation; Case, Shiller, and Thompson, 2012, for evidence from housing market; Smith, Suchanek, and Williams, 1998 and Haruvy, Lahav, and Noussair, 2007, for experimental evidence). Under infrequent portfolio adjustments, beliefs that are unstable due to extrapolation bias would result in less close alignment between beliefs and actual portfolio composition. Misalignment of current beliefs and current portfolio composition may also appear if households think that their beliefs are less stable and potentially biased. Another potential reason for the attenuation is paying limited attention either to changes in the stock market or to the evolution of one's own financial portfolio, both of which are consistent with rational inattention models in Woodford (2012) and Caplin and Dean (2015).<sup>26</sup>

While data do not allow for testing all these theories directly, we can uncover interesting evidence by analyzing heterogeneity in the attenuation by observable characteristics. The most robust finding is that the response of stock shares to beliefs and preferences varies over education groups. Recall that our sample has a relatively large group of more educated investors, so we can make relatively fine distinctions by educational attainment. Table 7 shows the results

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<sup>26</sup> In Appendix F, we examine the latter possibility by using the number of logons to Vanguard web accounts in the past 6 months as a measure of attention. We do not find strong evidence that attenuation is related to attentiveness as measured by checking accounts on the web. There are differences in the point estimates, but they are not large relative to their standard errors.

with the largest and most robust differences by educational attainment. The correlation of stock shares with expected returns is substantially stronger among those with MBA degree than the rest of the investors, at least when the survey measure of stock share is considered. The R-squared is about 20 percent, quite high for a cross-sectional regression. The difference in the estimated coefficient between the MBA graduates and the other groups is statistically significant. Though the small sample size of the MBA group makes the estimates noisier, the finding in Table 7 point to important differences in how preferences and beliefs translate to investment behavior: The most financially sophisticated investors place much more weight on their preferences and beliefs in determining their portfolio shares.

Table 7. Different Attenuation by Education:  
Stock Shares versus Cardinal Proxies for Preferences and Beliefs in Three Groups

	MBA		Other post-college		No post-college	
	Survey	Admin	Survey	Admin	Survey	Admin
$\hat{\mu}_i$	0.136*** (0.029)	0.050 (0.032)	0.052*** (0.014)	0.062*** (0.014)	0.055*** (0.012)	0.043*** (0.010)
$\hat{\sigma}_i$	0.009 (0.151)	-0.129 (0.165)	-0.098 (0.069)	-0.119* (0.063)	-0.064 (0.053)	-0.055 (0.047)
$\hat{\theta}_i$	0.040* (0.024)	0.028 (0.026)	0.023 (0.014)	0.033** (0.013)	0.031** (0.011)	-0.003 (0.009)
Constant	1.643 (2.163)	-1.348 (2.370)	1.416 (0.921)	1.165 (0.845)	0.910 (0.764)	0.488 (0.667)
Covariates	Y	Y	Y	Y	Y	Y
R <sup>2</sup>	0.206	0.175	0.037	0.040	0.043	0.030
N	287	287	1,538	1,538	2,589	2,589

Notes. Left-hand-side variables and covariates as in Table 5. Bootstrap standard errors in parentheses. \*, \*\*, and \*\*\* imply significance at 5%, 1% and 0.1% level, respectively.

What accounts for the heterogeneity of the weight that individuals put on their expectations and preferences in choosing their portfolio shares? We now explore two possible channels: stability of expectations and awareness of portfolio share.

*Stability of expectations.* If individuals are not confident of their expectations, they might put less weight on them in making decisions. We do not have a direct measure of confidence, but we can assess how malleable individual expectations are. In particular, we estimate how sensitive expectations are to recent stock market performance. Using the variation in the timing that the respondents filled in Survey 3, we first examine whether respondents' beliefs are affected by returns experienced during the week before the interview. About half of the respondents filled in the survey within one week after the invitation, but many did so over the next several weeks, often responding to reminders we sent out. We regressed the cardinal proxy of respondents' beliefs about mean returns  $\hat{\mu}_i$  on  $r_w$ , the stock returns within the week preceding the interview ( $r_w = (SNP_{t-1} - SNP_{t-8})/SNP_{t-8}$ , where  $SNP$  is the S&P 500 index and  $t$  is the day of survey). We also repeat the exercise with the raw survey response of mean returns,  $m_i$ , as the dependent variable. Appendix E gives details. Appendix Figure E1 shows how the distribution of response days compares aligns with the previous week's stock return. Appendix Table E1 gives the estimated relationship between mean and median expected stock returns from the survey and the previous week's realized S&P return.

For the entire sample, the regression of  $\hat{\mu}_i$  on  $r_w$  yields a slope coefficient estimate of 0.16 with a standard error of 0.056. (Regressing  $m_i$  to  $r_w$  returns a similar but less precise coefficient (0.18, significant only at 10 percent)). This estimate implies that respondents that experienced a one-percent point higher weekly return in the past week report 0.16 percentage point higher expected return. Thus, on average, expectations appear to respond relatively strongly to recent changes in stock prices. This effect is similar to those found in the literature (e.g., Greenwood and Shleifer, 2014, Hudomiet, Kezdi and Willis, 2011, and Hurd, Van Rooij and Winter, 2011). In contrast, we find that there is no such response among those with an MBA

degree. For the mean return, the estimated point response to lagged realized returns is negative. For the median, it is close to zero. For both specifications, the coefficient is insignificantly different from zero albeit with a large standard error. Hence, MBA graduates have more stable beliefs than most other investors in that they do not update beliefs in response to recent events. Thus, unstable beliefs may be an important reason for the attenuation. Indeed, those with unstable beliefs would require large and frequent portfolio reallocation if they acted on their beliefs. Doing so would require substantially more frequent adjustments than what conventional advice suggests, and that we observe in the data (see Figure 1B). Moreover, if investors expect their beliefs to change in the future they should not fully react to current changes in their beliefs.

*Awareness of portfolio share.* A second channel whereby households might down-weight their expectations and preferences in choosing their portfolios is awareness. Our finding in Figure 1D indicates that many respondents report very different stock share of their Vanguard accounts than the stock share implied by administrative data on their accounts. The correlation between the two measures is 0.64 in the entire sample; it is 0.66 among singles (the survey measure combines the accounts of spouses that we cannot identify in the administrative data). The corresponding correlations among MBA graduates are 0.71 and 0.88. These numbers indicate that MBA graduates are more aware their account holdings than most other investors.

*Discussion.* The central result of the papers is that individual-level expectations and preferences do matter for portfolio choice: stock shares vary across individuals with subjective mean and variance of returns and with risk tolerance in proportionately to what theory suggests. Yet, the response of portfolios to expectations and preferences is substantially attenuated. This section explores hypotheses that might account for this attenuation. It is much less pronounced for MBA graduates. MBAs also have more stable preferences and are more aware of their portfolio



composition. Putting these findings together provides circumstantial evidence for the sources of the attenuation. More financially sophisticated investors may put more weight on their subjective variables if have greater confidence in these assessments and greater confidence in their awareness of their portfolios.

#### **IV. Conclusion**

There is substantial heterogeneity in portfolio decisions across households. This paper uses a distinctive measurement and analytic strategy that combines high-quality measurement of portfolio shares, preferences about risk, and beliefs about returns in an attempt to explain this heterogeneity. The approach uses purposely-constructed measures to elicit measures of preferences and beliefs that have quantitative interpretations. This paper does find that risk preference and moments of the subjective returns distribution—both mean and variance—do have a role in understanding why portfolio choices differ. Relative to each other, the magnitudes of the coefficients on expected returns, perceived risk, and risk tolerance, are in proportion with the predictions of benchmark theories. That the survey measures of preference and belief do align with portfolio choices provides external validation of our approach to measuring them.

The size of the estimated associations of the risk and belief parameters is, nonetheless, substantially smaller in magnitude than benchmark theories would suggest. We call this finding the “attenuation puzzle.” Our methods produce risk and belief parameters that measure the precise, quantitative variables that should explain portfolio choice. Hence, the attenuation cannot be dismissed because the measures of preference and belief are only loosely related to what should drive portfolio choice. Moreover, the statistical procedure deals with measurement error in these parameters, which is the other most obvious source of such attenuation. The econometric procedure estimates the response error in the survey measures of preferences and beliefs and

produces individual-specific preference and belief parameters that are immune from the standard effects of using noisy estimates as regressors.

We argue that the selected nature of the sample is unlikely to account for the small magnitudes. The results are nearly identical for the subset of the sample who came to Vanguard via their employer's choices, so individual selection to use Vanguard as a provider is not driving the findings. Another explanation for the attenuated response could be background risk that would reduce stockholding given the perception of riskiness of stock returns per se. This source of attenuation, however, mostly affects the coefficient of our measure of perceived risk, and it should not lead to substantial bias in the estimates of the coefficients of risk tolerance or perceived expected returns.

Instead, we found evidence that those with an MBA degree, a group of presumably better educated investors, have portfolio composition that is more aligned with their beliefs. Their beliefs are not responsive to recent history of prices, in contrast with a strong reaction, on average, among other investors. MBA graduates are more aware of their account holdings, as evidenced by smaller discrepancy between the portfolio composition they report on the survey and the one implied by administrative data from their accounts. Thus, unstable beliefs and awareness of one's portfolio may be important explanations behind the attenuation puzzle. Of course, other differences between the better educated MBA graduates and other investors, that are unmeasured in our data, may also play an important role. In general, the attenuation puzzle is consistent with most decision makers mixing their own risk preference and beliefs about stock market returns with the preferences and beliefs of the representative consumer.

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