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**Product Differentiation and Demand Elasticity**

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# PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

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## ABSTRACT

This paper argues that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to size of market. Despite Chamberlin's view, monopolistic competitors are price takers, even though each firm's product has no perfect substitute. There is a difference between perfect competition with product homogeneity and perfect competition with differentiated products, however. Advertising can pay off with differentiated products because products have separate identities—and price depends on quality—even though firms are price takers for any given quality. A differentiated oligopoly may resemble monopolistic competition a la Chamberlin in some ways.

**JEL Classifications:** D41, D43.

**Key Words:** Monopolistic Competition, Perfect Competition, Product Differentiation.

## PRODUCT DIFFERENTIATION AND DEMAND ELASTICITY

This paper argues that product differentiation is compatible with perfect competition under free entry and exit and small firm size relative to size of market. Under the conditions given by Chamberlin [1965] in his classic treatise on monopolistic competition, firms would be price takers and perfect competition would prevail, although with a key exception relating to advertising. Similar results have been derived before—for example, by Fradera [1986] and Rosen [1974]—but the approach here is simpler, shorter, and freer of restrictive assumptions. It focuses on the key issue of demand elasticity. Despite the widespread view in economics that monopolistic competitors face downward-sloping demand and produce with excess capacity and sub-optimal firm size, the existence of many imperfect substitutes for a product is enough to turn its supplier into a price taker. For an industry to have these properties, we need a number of competitors that is not so large that each firm is a *de facto* price taker, but also not so small that firms are able to earn positive economic profit.

To show that firms facing many competitors are price takers, I first note that monopolistic competition implies many firms in an industry—the result of free entry and exit and large market size relative to the output that minimizes average cost for any firm. Each firm supplies a single product, and as in perfect competition, has an insignificant share of industry output. Buyers of these products are assumed to maximize utility. Firms maximize profit and reach a Nash equilibrium, in which no firm can gain profit by changing its price if prices of other firms remain unchanged. Marginal costs and prices are positive for firms that survive. Firms supply products that are ‘close’ but not perfect substitutes, the factor that distinguishes monopolistic from perfect competition. Product  $Y$  is defined to be a ‘close’ substitute for product  $X$  if and only if the cross-price elasticity between the two that arises when the price of  $X$  changes, with other prices held constant, is greater than or equal to some positive lower bound. Otherwise  $X$  does not really compete with  $Y$ .

Let  $X$  be a differentiated product in an industry called the  $X$  industry that operates under monopolistic competition. Let  $P_x$  and  $x$  be the price and quantity of  $X$  and  $\varepsilon_x$  be the own-price elasticity of demand for  $X$ .

Suppose that  $P_x$  changes by  $dP_x$ , with all other prices in the economy remaining constant. If  $dP_x/P_x$  is numerically small,  $\varepsilon_x$  approximately equals  $dx/x$  divided by  $-(dP_x/P_x)$ , where  $dx$  is the change in quantity demanded of  $X$ . Let  $I^*$  be the income of the economy in which the  $X$  industry operates and  $E_x$  be the expenditure on all products that are neither substitutes for nor complements with  $X$ . If  $I = I^* - E_x$ ,  $I$  is the sum of expenditures on  $X$  and on products that are either substitutes for or complements with  $X$ . The economy is assumed to be large enough that  $I^*$  is independent of changes in  $P_x$  and, by definition,  $E_x$  is unaffected by such changes. Thus  $I$  remains constant when  $P_x$  changes.

However, a change in  $P_x$  does cause changes in  $x$  and in each other output in  $I$ . Let  $S_x = P_x x / I$  be the share of  $X$  in  $I$ , and when  $P_x$  changes with other prices held constant, let  $\varepsilon_{Ax}$  be the share-weighted average cross-price elasticity of demand over the products that are substitutes for and complements with  $X$ . Each product's cross-price elasticity equals the percentage change in its quantity divided by the percentage change in  $P_x$ . The shares in question are the shares in  $I$  of expenditures on each substitute and complement, and the sum of these shares equals  $(1 - S_x)$ , while the sum of each share times that product's cross-price elasticity equals  $(1 - S_x)\varepsilon_{Ax}$ . Straightforward calculation yields  $S_x(1 - \varepsilon_x) + (1 - S_x)\varepsilon_{Ax} = 0$ , since a change in  $P_x$  does not affect  $I$ . Re-arranging this gives:

$$\varepsilon_x = 1 + [(1 - S_x)/S_x]\varepsilon_{Ax}. \quad (1)$$

Suppose that only a few firms are competing in the  $X$  industry initially and earning positive economic profits, but that these profits attract further entry until equilibrium is reached with zero economic profits. In the process,  $S_x$  tends to zero, and  $(1 - S_x)/S_x$  tends to infinity. At the end of the paper, we shall ask how small  $S_x$  has to be in practice for a firm to be a price taker. Now the task is to show that as  $S_x$  tends to zero,  $\varepsilon_x$  tends to infinity. Here the key is to show first that, as  $S_x$  tends to zero, either  $\varepsilon_{Ax}$  remains bounded above zero or  $\varepsilon_x$  tends to infinity. However, if  $\varepsilon_{Ax}$  remains bounded above zero as  $S_x$  tends to zero—that is, if there exists a  $B > 0$  such that  $\varepsilon_{Ax} \geq B$ —it is clear that  $\varepsilon_x$  also tends to infinity as  $S_x$  tends to zero.

To show the first result, let  $I_x$  be total expenditure on the  $X$  industry, as well as  $X$  industry output value, and let  $I_{nx}$  be the total expenditure on substitutes for and complements with  $X$  that are not in the  $X$  industry. Thus  $I = I_x + I_{nx}$ . If  $S_x^x = P_x x / I_x$  is the share of  $X$  in  $I_x$ , then  $S_x^x = S_x(I/I_x) \geq S_x$ , with the strict inequality holding

when  $I_{nx} > 0$ . When  $S_x^x$  tends to zero, the same will be true of  $S_x$ . Here  $\varepsilon_{Ax}$  is a sum of terms, each of which equals the share of a product in  $I$  times that product's cross-price elasticity divided by  $(1 - S_x)$ . The cross-price elasticity equals the percentage change in the product's output divided by the percentage change in  $P_x$ . We can write  $\varepsilon_{Ax}$  as  $\varepsilon_{Ax} = \varepsilon_{Ax}^x + \varepsilon_{Ax}^{nx}$  where  $\varepsilon_{Ax}^x$  is this sum over all products in the  $X$  industry except  $X$ , and  $\varepsilon_{Ax}^{nx}$  is this sum over products that are substitutes for or complements with  $X$ , but which are outside the  $X$  industry.

As entry into the  $X$  industry occurs, consider what happens to  $dI_x/I_x$  when  $P_x$  falls, with prices of other products held constant. First note that as  $S_x^x$  and  $S_x$  tend to zero, either  $\varepsilon_x$  tends to infinity or  $\varepsilon_x$  remains finite, in which case  $dx/x$  is bounded relative to  $-(dP_x/P_x)$  for all  $dP_x$  sufficiently close to zero. Suppose that the latter is true. Then  $dI_x/P_{xX}$  is bounded above since the fall in  $P_x$  causes quantities demanded of substitute products to fall. That is, if  $dx/x \leq -A(dP_x/P_x)$  for some finite and positive  $A$ ,  $dI_x/P_{xX} < (1 - A)(dP_x/P_x)$  must hold.

In addition,  $dI_x/P_{xX}$  is bounded below if the fall in  $P_x$  is sufficiently small. Otherwise, as  $S_x^x$  tends to zero,  $dI_x/P_{xX}$  will tend to minus infinity for any given small decrease in  $P_x$ , and  $dI_x/P_{xX}$  will therefore tend to minus infinity as  $S_x^x$  and  $dP_x$  tend to zero. Suppose that  $dI_x/P_{xX}$  is a continuous function of  $S_x^x$  and of  $dP_x$  everywhere in some neighborhood of  $S_x^x = dP_x = 0$ . Then if  $dP_x = 0$  and  $S_x^x$  is small enough,  $dI_x$  will be negative. With no changes in prices or incomes, a transfer of demand will occur from products in the  $X$  industry to products outside this industry. However, if buyers were already maximizing utility, there would be no need for such a transfer. Thus the assumptions of utility maximization and of continuity imply that  $dI_x/P_{xX}$  remains bounded below as  $S_x^x$  and  $S_x$  tend to zero.

Since  $dI_x/I_x = S_x^x[dI_x/P_{xX}]$  and  $dI_x/P_{xX}$  is bounded above and below,  $dI_x/I_x$  tends to zero as  $S_x^x$  and  $S_x$  tend to zero. If  $I_{nx}$  tends to zero, note that  $\varepsilon_{Ax}^{nx}$  also tends to zero—since in the limit there are no complements with or substitutes for  $X$  outside the  $X$  industry—and  $\varepsilon_{Ax}$  tends to  $\varepsilon_{Ax}^x$ . If  $I_{nx}$  is positive in the limit, then  $dI_{nx}/I_{nx}$  tends to zero, since in the limit:

$$0 = dI/I = (I_x/I)(dI_x/I_x) + (I_{nx}/I)(dI_{nx}/I_{nx}) = (I_{nx}/I)(dI_{nx}/I_{nx}) \quad (2).$$

when  $P_x$  falls. However,  $(dI_{nx}/I_{nx})$  divided by  $(dP_x/P_x)$  tends to  $\varepsilon_{Ax}^{nx}(I/I_{nx})$ . Therefore, as  $S_x^x$  and  $S_x$  tend to zero,  $\varepsilon_{Ax}^{nx}$  also tends to zero if  $dx/x$  remains bounded relative to  $-(dP_x/P_x)$ . As a result,  $\varepsilon_{Ax}$  again tends to  $\varepsilon_{Ax}^x$ .

The final step is to show that  $\varepsilon_x$  tends to infinity when  $\varepsilon_{Ax}$  tends to  $\varepsilon^x_{Ax}$ . By assumption, all products in the  $X$  industry are ‘close’ substitutes for  $X$ . The cross-price elasticity between  $X$  and any other product in the  $X$  industry that arises when  $P_x$  changes cannot be arbitrarily small, but must instead be no less than some positive value, say  $B > 0$ . As a weighted average of these cross-price elasticities,  $\varepsilon^x_{Ax}$  must tend to a value that is no less than  $B$  as  $S^x_x$  and  $S_x$  tend to zero. Since  $\varepsilon_{Ax}$  tends to  $\varepsilon^x_{Ax}$ , the same must be true of  $\varepsilon_{Ax}$ . As a result,  $\varepsilon_x$  tends to infinity, which is therefore the only possible limiting value for  $\varepsilon_x$ . By making  $S^x_x$  and  $S_x$  small enough,  $\varepsilon_x$  can be made as large as desired.

In fact, the ‘ $X$  industry’ is a construct defined to consist of  $X$  and of all products that are ‘close’ substitutes for  $X$ , in the sense that each of the relevant cross-price elasticities when  $P_x$  changes is no less than  $B$ . The definition of the ‘ $X$ -industry’ will depend on the value of  $B$  selected, but the values of  $\varepsilon_x$ ,  $S_x$ , and  $\varepsilon_{Ax}$  are clearly independent of  $B$ . To a degree, the choice of  $B$  is arbitrary, but the choice of industry boundaries is always somewhat arbitrary. For the selected value of  $B$ , suppose that the set of all ‘close’ substitutes for  $X$  forms an equivalence class. That is, suppose that when product  $X$  is a ‘close’ substitute for product  $Y$ , then  $Y$  is a ‘close’ substitute for  $X$ , and that when  $X$  is a ‘close’ substitute for  $Y$  and  $Y$  is a ‘close’ substitute for  $Z$ , then  $X$  is also a ‘close’ substitute for  $Z$ . In this case, the  $X$  industry consists of all firms whose products are ‘close’ substitutes for  $X$ , which implies that these products are all ‘close’ substitutes for one another as well. Each firm in the  $X$  industry so defined is a price taker when the industry consists of many products, with each firm having a small share of industry output value. However, we do not need to assume that the set of all ‘close’ substitutes for  $X$  forms an equivalence class to show that the supplier of  $X$  is a price taker.

How small does  $S_x$  have to be in order for the supplier of  $X$  to be a *de facto* price taker? Suppose that  $S_x = .03$  and  $\varepsilon_{Ax} = .3$ . If the difference between  $\varepsilon_{Ax}$  and  $\varepsilon^x_{Ax}$  can be ignored—since one tends to the other— $\varepsilon_{Ax}$  is the share-weighted average cross-price elasticity over the  $X$  industry, which would be infinitely large if all products in this industry were perfect substitutes. With  $\varepsilon_{Ax} = .3$ , a 10% decrease in  $P_x$  would lower the demand of an average competitor by 3%. If  $I_x$  is 70% of  $I$  for the value of  $B$  selected, the share,  $S^x_x$ , of  $X$  in  $I_x$  is about  $1.43S_x = .0429$ , implying an industry with 23 suppliers if  $S^x_x$  is an average share for this industry. In this case,  $\varepsilon_x$

= 9.7. If the supplier of  $X$  raised its price by 5%, it would lose nearly half its market. Such a firm has little room for price maneuver and is a *de facto* price taker.

However, if  $S_x$  were twice as large and  $I_x$  were again 70% of  $I$ , the share,  $S_x^x$ , of  $X$  in  $X$  industry output value would be .0858, implying an industry with 11 or 12 suppliers if  $S_x^x$  is about average for this industry. Then  $\varepsilon_x$  equals 4.7, and the industry is a differentiated oligopoly. Nevertheless, if firms in this industry are unable to earn positive economic profit, the industry may behave in some ways like a monopolistically competitive industry in Chamberlin. For this behavior, we need a number of competitors that is not so large that each firm is a *de facto* price taker, but also not so small that firms are able to earn positive economic profit.

It follows that Chamberlin's monopolistic competition with many competitors is a type of perfect competition, although with a key exception. When products are differentiated, they and the firms that supply them have separate identities and can be distinguished from one another. It is therefore possible to advertise a specific firm's product successfully if the advertising leads potential customers to believe that it has a higher quality than they had previously perceived. For that quality, the firm is still a price taker, however.

While market failure can always result from too few competitors and entry barriers, it does not result from product differentiation with many competitors. Chamberlin's conclusion that the demand for  $X$  is downward sloping requires the share-weighted average cross-price elasticity,  $\varepsilon_{Ax}$ , to tend to zero as the share of  $X$  in  $I$  tends to zero, but either this does not happen or  $\varepsilon_x$  tends to infinity anyway.

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