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**The Role of Model Specification in
Estimating Health Care Demand**

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The role of model specification in estimating health care demand

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Abstract

Zero inflation and over-dispersion issues can significantly affect the predicted probabilities as well as lead to unreliable estimations in count data models. This paper investigates whether considering this issue for German Socioeconomic Panel (1984-1995), used by Riphahn et al (2003), provides any evidence of misspecification in their estimated models for the adverse selection and moral hazard effects. The paper has the following contributions: first, it shows that estimated parameters for adverse selection and moral hazard effects are sensitive to the model choice; second, the random effects panel data as well as standard pooled data models do not provide reliable estimates for health care demand (doctor visits); third, it shows that by appropriately accounting for zero inflation and over-dispersion there is no evidence of adverse selection behaviour and that moral hazard plays a positive and significant role for the number of doctor visits. These results are robust for both males and females' subsamples as well as for the full data sample.

Key words: Over-dispersion, zero-inflated distribution, adverse selection, moral hazard

1. Introduction

The demand for health care, as measured by the number of visits to doctors or hospitals, is very important in determining the pricing of insurance policies. The asymmetric information that characterizes the interaction between the individuals and the insurance providers may induce adverse selection and moral hazard problems that can affect both the demand for health care and the type of insurance an individual buys. It is worth mentioning that adverse selection occurs when the insured has more information than the insurance company. As Chiappori and Salani (2000) argue, adverse selection can encourage a high risk individual to choose higher coverage. Following Cameron *et al.* (1988) and Riphahn and *et al* (2003) this can be explained by finding a positive correlation between the choice of insurance and demand for health care. Moral hazard occurs when due to a lower cost, an insured individual demands more health care than an identical uninsured one. Geil *et al.* (1997) study the effect of moral hazard on

hospital demand by using German data. He does not find a statistically significant coefficient for moral hazard, while in contrast, Cameron *et al.* (1988) find evidence of it in Australian data.

To ensure that the demand for health care is optimal, it becomes very important to have a system that provides optimal pricing; therefore, an appropriate measure of the demand for health care is required. Since the number of visits to doctors or hospitals in the health demand function are integers, methodologically, count data models are suitable choices to model these visits. Several researchers have investigated health care demand by focusing on different data sets. For example, Pohlmeier and Ulrich (1995) as well as Cameron *et al.* (1988) use cross-sectional data. Geil *et al.* (1997) uses a single equation random effect model to improve the fitness of the model. Based on European data, Bago d'Uva and Jones (2009) estimate a pooled panel for doctor visits using a latent hurdle model. Their study provides a positive income effect on visiting doctors. Using data from Italy, Atella and Deb (2008) found that primary care doctors and specialists are substitutes for both private and public sectors. Marvasti's study (2014) is based on "a panel of the U.S. Census Bureau division data for annual average number of physician office visits during the 1987–1997 period". He estimates with panel data a simultaneous equations model for both supply and demand of doctor visits. He finds that when private and public insurance coverages are incorporated in the model, the demand for doctors is neither income elastic nor price elastic. Also, having easy access to doctors in divisions with high densities of doctors increases the number of visits to doctors.

Rephahn *et al.* (2003), who base their model on Cameron *et al.* (1988), use a bivariate random effects model to estimate simultaneously the demand for doctor and hospital visits using the German Socioeconomic Panel (GSOEP, 1984–1995) data. One of the shortcomings of their paper is that they use models that do not account for the fact that the data set has around 90% and 35% zeros for the number of doctor visits and hospital visits, respectively. This means that they do not take into account the possibility of the existence of over-dispersion in the data, which in the end affects the estimation of the predicted probabilities. Rephahn and *et al.* (2003) used a model with random effects by mixing a Poisson distribution with log normal distributions. However, other generalized versions of the standard distributions as well as the family of zero-inflated models might be better options for explaining the existence of over-dispersion in the data (see Harris *et al.* (2014), and Hilb (2011)).

In this paper we test if the results of Rephahn and *et al.* (2003) are valid by employing both standard distributions that ignore the zero-inflated feature of the data and methods that account for the zero-inflated aspect of the data. We show that by accounting for the zero-inflated feature of the data a significantly

different impact is found on the adverse selection and moral hazard measures, a result that has different implications on health-care demand measures.

Researchers use different versions of zero-inflated models based on their efficiency; for example, zero-inflated Poisson (ZIP), a mixture of Poisson distribution and a distribution with a point mass at zero is introduced by Lambert (1992). He uses this model to investigate the defects in a manufacturing process that has a big share of zeros in the sample. Zero-inflated models are more used in ecology and have been developed by Ridout *et al* (1998), Martin *et al* (2005) and Kuhnert *et al* (2005) for explaining different kinds of zeros; those that are structural as well as those that depend on the study. An application of Bayesian analysis for zero-inflated models is discussed in Angers and Biswas (2003). Also, more references and studies related to the development of zero-inflated models for continuous data are discussed in Martin *et al* (2005). Greene (1994) considers the zero-inflated negative binomial (ZINB). This model is used by Neal and Gaher (2006) to study drug use issues among college students. Gupta *et al* (1996) and Famoye (2006) applies zero-inflated Generalize Poisson for the study of frequentist setting. Cohen (1960), Johnson *et al* (2005) use zero-inflated models when zeros are underscored. Some authors proposed models that consider other positive values that are larger than what is explained by Poisson distribution. For example, Melkersson and Rooth (2000) introduce a parent Poisson and NB2 to explain inflated zero and two. However, they assume that measurement errors is the main cause of under recording values other than zero and two. Li *et al* (2003) introduce a more flexible model for this situation, while Carroll *et al* (2006), Buonaccorsi (2010) and Chen *et al* (2011) provide more details related to recent researches in measurement error with emphasis on Poisson model.

As will be discussed in following sections, recently various extensions of zero-inflated models have been emphasised by Haris and *et al* (2014) and are incorporated in STATA. More details about some of these models are discussed in Hilb (2011). While, STATA is not able to provide panel data estimates for zero-inflated models, LIMDEP is able to estimate fixed effect and random effect models in this context.

The outline of the paper is as follows. Section 2 discusses different methodologies that are employed. Section 3 describes the data used in the analysis, Section 4 provides the results of different model specifications, Section 5 evaluates the fit of the employed models, Section 6 does robustness checks and Section 7 concludes.

2. Methodology

Motivation – Discussion of Over Dispersion

In what follows we discuss the importance of properly accounting for over dispersion when it is present in count data models such as the ones used to model doctor and hospital visits. The Poisson model is a widely used model in this context; however, one of the shortcomings of this model is that its mean and variance are equal. This feature of the model is not helpful when variance exceeds its mean (or in other words we have over dispersion). Over dispersion is generally caused by a positive correlation between responses or by an excess variation between response probabilities or between counts. Such correlations can come from a clustering effect that may lead to underestimation of standard errors of the estimated coefficient vector. In this context, some variables might be statistically significant while in reality they are not, which affects the inference of the model's parameters. For count models, over-dispersion can be recognized if the value of the Pearson statistic (divided by degree of freedom) is larger than 1. If this measure is greater than 2 the over-dispersion is severe, and an adjustment to standard errors may be required. The over-dispersion may be present when the model might omit an important explanatory variable, the data might contain outliers or the predictor needs to be transformed to a logarithm or other scales¹. In linear models, by using robust variance estimators as well as bootstrapping or jackknifing standard errors one can deal with over-dispersion.

In what follows we present how over-dispersion can be taken into account for count data models through mixing the Poisson distribution with other distributions.

2.1 Count data models

A Poisson model (with $E(Y_i) = V(Y_i) = \mu_i$), cannot deal with the over-dispersion due to the equality of mean and variance. To make it more flexible the model can be augmented with other distributions. This is done by relating its mean to an individual unobserved effect (u_i). Different extensions of the Poisson can be obtained depending on how the distribution for u_i is specified. Appendix B provides an extensive discussion about these extensions and in Table B1 and B2 the reader finds a list of distributions that are used in the proposed analysis as robustness checks of different model specifications.

¹ See Hilb, 2011, chapter 12.

For any specified distribution for u_i it can be shown that it produces a new distribution with higher variance than the Poisson model. For instance, in the case u_i has a Gamma distribution, with mean 1 and variance $\alpha = \frac{1}{\theta}$, the mixture of Poisson-Gamma is called Negative Binomial 2 (NB2) which has mean μ_i and conditional variance $\mu_i + \alpha\mu_i^2$. The Negative Binomial 1 (NB1) model is obtained by assuming $\theta_i = \theta\mu_i$. In NB1 and NB2, α refers to heterogeneity parameter. In addition, when $\alpha = 0$ both distributions will be identical to the Poisson.

Besides, the NB1 and NB2, we consider the power negative binomial (NB-P) distribution, which has an additional parameter, p , see Greene (2008). For $p = 1$ and $p = 2$, this distribution collapses to NB1 and NB2, respectively.

In NB2, the dispersion parameter (α) is constant. The NB2 can be generalized to a heterogeneous NB2 model that allows the dispersion parameter to vary across observations: $\alpha = \exp(z_i\gamma)$. This gives more flexibility to the dispersion parameter to capture most of the variation in the data. One can find other generalized versions of NB models as the one introduced by Famoye (1995).

Further, we consider univariate generalized Warning distribution or the beta Negative Binomial model, which is another extension of Negative Binomial model that was introduced by Irwin (1968). This model has more flexibility for specifying the unobservable heterogeneity. In other words, as Hilbe (2014) explains “unobserved heterogeneity can be separated from the internal factors of each individual’s characteristics and external factors (covariates) that may affect the variability of data”(see Hilbe 2011).

Moreover, we also employ a Generalized Poisson (GP) model that can accommodate both over-dispersion and under-dispersion.

2.2 Zero-inflated count models

Another source of over-dispersion is the presence of a large number of zeros in the data. In other words, when there are more zeros than the expected number of zeros under a specified distribution, then the data may be over-dispersed. Zero-inflated Poisson (ZIP) model and zero-inflated negative binomial (ZINB) models adjust for excessive zeros in the response. Recently, Hilbe and *et al* (2014) discuss different versions of zero-inflated models.

Zero-inflated models consider two distinct sources of zero outcomes. One source is generated from individuals who do not enter the counting process, the other from those who do enter the counting process but result in zero outcomes. As an example, suppose that we have data related to the number of visits to

doctors. However, we might have individuals in our data that they do not have any doctor. Then the probability of observing a zero outcome equals the probability that an individual is in the group that does not have a doctor ($B(0)$) plus the probability that the individual is in the other group multiplies the probability he/she does not see the doctor $\{1 - B(0)\}Pr(0)$, where $Pr(0)$ is the counting process that produces the zero; see Hilbe and Greene (2008). In this particular case, the model has two parts: The probability of a zero outcome for the system and the probability of a nonzero count².

The obtained mixture gives more power for the model to explain the sources of over-dispersion in the data. Table 2B of the Appendix B presents different zero-inflated distributions that are used as robustness checks for our analysis. For the purpose of comparing the suitability of a zero-inflated distribution against its standard distribution, a test introduced by Vuong (1989) is applied, which is a likelihood ratio based test for selecting a specific model among non-nested models. A significant positive Vuong statistic leads to accepting a zero-inflated model, while a significant negative value for the test rejects a zero-inflated model in favour of its standard model. A non-significant Vuong statistic indicates no preference for either models. As an additional check we look at the predictability of different model specifications.

3. Data description:

We use the same data as in Rephahn and *et al* (2003) that is “the first twelve annual waves (1984 through 1995) of the German Socioeconomic Panel (GSOEP) which surveys a representative sample of East and West German households”. The data set is downloadable from the web site of Journal of Applied Econometrics³. The data is restricted to individuals aged between 25 and 65. Table 1 presents the descriptive statistics of the dependent variable by gender⁴. Dependent variables are defined as “the number of visits to a doctor within the last quarter prior to the survey, and the number of inpatient hospital visits with at least one night spent in the hospital within a given calendar year”.

Table 1 shows the presence of many zeros in both hospital visits and doctor visits for both genders. Around 92% and 44% of males did not visit a hospital and a doctor, respectively. For females, the shares of zero hospital and doctor visit are around 90% and 30%, respectively. The abundance of zeros in both kinds of visits suggests that zero-inflated distributions might be better options rather than their standard

² Stata gives this probability using the command: predict fk, pr(k)

³ See: <http://qed.econ.queensu.ca/jae/>

⁴ For more detail about the data see Table A1 as well as Rephahn and *et al* (2003).

versions for the purpose of examining doctor and hospital demands. Since the frequency of zeros for doctor visits is less than for hospital visits, this paper focuses on the demand equation for doctor visits. If the results for this equation confirm the superiority of zero-inflated distributions over their standard versions, the results can be extended to demand for hospital visits as well. Among the explanatory variables, two different dummy variables for the types of insurance are considered: whether an individual has public insurance or not, and, if yes whether he or she has an add-on insurance policy. All the explanatory variables are the same as Rephahn *et al* (2003); see table A3 in the appendix.

Table 1: Dependent variables: Descriptive statistics

| Value | (Share of total observation, %) | | | |
|-------------|---------------------------------|---------|--------------|---------|
| | Hospital visit | | Doctor Visit | |
| | Males | Females | Males | Females |
| 0 | 92.21 | 90.18 | 44.05 | 29.51 |
| 1 | 6.18 | 7.88 | 13.82 | 13.17 |
| 2 | 1.09 | 1.28 | 11.63 | 13.42 |
| 3 | 0.15 | 0.27 | 8.48 | 11.49 |
| 04-Sep | 0.21 | 0.25 | 15.29 | 21.83 |
| 10 and more | 0.16 | 0.14 | 6.73 | 10.58 |
| Mean | 0.128 | 0.15 | 2.63 | 3.79 |
| Std dev. | 0.93 | 0.83 | 5.21 | 6.11 |
| Median | 0 | 0 | 1 | 2 |
| N | 14243 | 13083 | 14243 | 13083 |

Source, German Socioeconomic Panel (1984-1995)

4. Discussion of the Results

4.1 Panel Data Models

Riphahn *et al* (2003) did not pay attention to the zero-inflated nature of the analyzed data. Consequently, by properly considering this data issue one can provide a more robust and reliable estimated parameters for add-on and public insurance variables.

On the demand side, Rephahn and *et al* (2003) assume a bivariate model for demands for doctor and hospital visits. These demands follow a Poisson distribution, and the unobservable heterogeneity and error terms follow lognormal and bivariate normal distributions, respectively:

$$y_{itg} \sim Po(\mu_{itg}) \quad g = 1,2 \text{ (with 1 for doctor visits and 2 for hospital visits)}$$

$$\ln(\mu_{it1}) = \beta' x_{it1} + u_{i1} + \varepsilon_{it1}; u_{i1} \sim N(0, \sigma_{u1}^2); (\varepsilon_{it1}, \varepsilon_{it2}) \sim N_2(0, 0, \sigma_{\varepsilon1}^2, \sigma_{\varepsilon2}^2, \rho); E[u_{ig}u_{jh}] = 0 \text{ if } i \neq j \vee g \neq h$$

$$\ln(\mu_{it2}) = \beta' x_{it2} + u_{i2} + \varepsilon_{it2}; u_{i2} \sim N(0, \sigma_{u2}^2); E[\varepsilon_{itg}u_{jsh}] = 0 \quad \forall i, t, g, j, h; E[\varepsilon_{itg}\varepsilon_{jsh}] = 0 \text{ if } t \neq s \vee i \neq j \vee g \neq h$$

To integrate out the unobserved heterogeneity u_{ig} , an outer integral based on Gauss- Hermite approximation was used, while to integrate out the distribution of cross-equation errors (ε_{itg}) an inner integral based on a modified Gauss-Legendre approach was applied. With this specification, Rephahn *et al* (2003) found that the add-on and public insurance coefficients in both demand equations are statistically insignificant and also found that the coefficient for males' hospital demand is negative.

In Germany, public insurance is mandatory for some specific groups of people with income less than a cut off, while for others it is not and they can choose private insurance. Also, add-on insurance is optional insurance that an individual can add to his or her public or private insurance to cover some other costs. Rephahn *et al* (2003) use this variable to test the adverse selection hypothesis. They found that the coefficients of add-ons in demands for doctor visits and hospital visits are positive but only statistically significant in the case of males' hospital demand. Also they found that self-employed females and males have fewer visits to doctors than other employees.

Rephahn *et al* (2003) estimate a simple random effect model for doctor visits by considering a Gaussian distribution for the heterogeneous term. For zero-inflated data the standard distributions are not suitable and alternative distributions (Gamma for example) that account for this particular issue with the data might be better options. Table 2 shows the doctor visits' results for females and males when both Gamma and Gaussian distributions are used. Based on AIC and BIC criteria, it can be seen that Gamma distribution is the better choice for the data. Also, with Gamma distribution the coefficient of public insurance is positive and statistically significant for both females and males while in the case of Gaussian distribution both of them are positive but insignificant. For both models, the coefficient of add-on is positive but not significant.

The results show that, although it seems that Gaussian distribution is more flexible than Gamma, NB2 (a mixture of Poisson and Gamma) is the better option for this data.

Table 2: Random effect model with Gaussian and Gamma distributions for the unobserved heterogeneity term

| | Males | | Females | |
|--------------|------------|------------|------------|------------|
| | RE Normal | RE Gamma | RE Normal | RE Gamma |
| Docvis | | | | |
| Age | -0.0299* | -0.0315*** | -0.0213 | -0.0208*** |
| | (0.0180) | (0.00815) | (0.0155) | (0.00686) |
| age2 | 0.520** | 0.509*** | 0.323* | 0.311*** |
| | (0.207) | (0.0934) | (0.176) | (0.0766) |
| Hsat | -0.182*** | -0.181*** | -0.143*** | -0.143*** |
| | (0.00971) | (0.00316) | (0.00766) | (0.00285) |
| Public | 0.106 | 0.103*** | 0.0638 | 0.0690* |
| | (0.0844) | (0.0388) | (0.0733) | (0.0360) |
| Addon | -0.0334 | -0.0340 | -0.0260 | -0.0317 |
| | (0.103) | (0.0535) | (0.0897) | (0.0456) |
| Constant | 2.093*** | 2.662*** | 2.041*** | 2.433*** |
| | (0.404) | (0.195) | (0.345) | (0.182) |
| Insig2u | 0.0138 | | -0.248*** | |
| | (0.0393) | | (0.0391) | |
| lnalpha | | -0.00860 | | -0.277*** |
| | | (0.0293) | | (0.0286) |
| Observations | 14243 | 14243 | 13083 | 13083 |
| AIC | 65802.1774 | 65713.7349 | 70856.0950 | 70728.1257 |
| BIC | 65976.1498 | 65887.7074 | 71028.1136 | 70900.1443 |
| Log lik. | -32878.1 | -32833.9 | -35405.0 | -35341.1 |

Standard errors in parentheses

Source, German Socioeconomic Panel (1984-1995)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.2 Standard count models:

Following Greene (2008), pooled data is used for the purpose of selecting the best model between standard models. Table 3 presents the results for all the estimated models based on standard distributions introduced in Table B1. The following models are Applied: Poisson, Negative Binomial 1 (NB1), Negative Binomial 2 (NB2), Generalized Poisson (GPoisson), Negative Binomial Famoye (NB Famoye) and Negative Binomial Waring .(NB Waring). –.

The results for Poisson, NB1 and NB2 are the same as in Greene (2008). Based on the dispersion criteria which has a high dispersion value of 6.67, Poisson distribution is not suitable. NB2 defeats Poisson by reducing considerably the value of dispersion to 1.99. This can also be confirmed by $\ln(\alpha) = 0.561$

which measures the logarithm of the dispersion parameter (α) and based on the likelihood ratio test is statistically significant. The same conclusion is obtained when other distributions are used: NB1, Generalized Poisson, NB Famoye and NB. The model selection is based on AIC and BIC criteria. Both criteria suggest that the NB Waring model is the best model for these data. It is followed by the Generalized Poisson, NB1, NB2, and NB Famoye. By looking at the estimated parameters for add-on, it can be seen that this parameter is positive and statistically significant based on NB Waring model. In addition this parameter is positive in all models but only significant in NB1 and Generalized Poisson and NB Waring models. The estimated parameter for public insurance is positive in all the models but statistically insignificant. Table A2 in the appendix A reports all the estimated parameters.

Table A3 in the appendix A, shows the same results for females. The results for dispersion and the ranking of the best models are the same as for males. Still, NB Waring is the best one and NB2 is in the second rank. The only difference is observed in add-on's estimated parameter. This parameter is positive, and, as for males, statistically significant for NB1 and generalized Poisson but not for NB Waring. For public insurance, all the models provide positive values but only in the Poisson model, which is a completely unreliable model, is it statistically significant. This can be viewed as evidence of over-dispersion, leading to underestimation of standard errors, making the coefficient statistically significant.

To conclude this analysis, we can state that even if we ignore the zero-inflated nature of the data, we can designate NB family and Generalized Poisson as better choices than the simple Poisson model.

Finally, Table 4 provides results for heterogeneous NB2. For this model, all the explanatory variables are used to explain its dispersion parameter. The investigation of the suitability of heterogeneous NB2 based on AIC and BIC criteria indicates that this model is better than the simple Poisson for both males and females. Also, based on AIC criteria, this model has the lowest value compared to other specifications. In this model only the coefficient of public insurance for females is statistically significant.

Table 3: Standard Distributions for Doctor Visit

| | Poisson | NB1 | NB2 | GPossion | NBFamoy | GNBWaring |
|--------------|---------------------|----------------------|----------------------|----------------------|-----------------------|----------------------|
| Docvis | | | | | | |
| Public | 0.100 (0.0702) | 0.0607 (0.0539) | 0.0934 (0.0635) | 0.0595 (0.0549) | 0.0934 (0.0635) | 0.0578 (0.0577) |
| Addon | 0.0666 (0.102) | 0.139* (0.0777) | 0.0551 (0.0948) | 0.144* (0.0791) | 0.0551 (0.0948) | 0.154* (0.0844) |
| Constant | 2.771*** (0.336) | 2.776*** (0.254) | 3.149*** (0.329) | 2.780*** (0.258) | 3.710*** (0.330) | 2.929*** (0.273) |
| Lndelta | | 1.581*** (0.0365) | | | | |
| Lnalpha | | | 0.561*** (0.0270) | | | |
| Atanhdelta | | | | 0.726*** (0.0115) | | |
| lnphim1 | | | | | -17.76*** (3.253) | |
| Lntheta | | | | | -0.561*** (0.0270) | |
| lnrhom2 | | | | | | 0.783*** (0.0981) |
| Lnk | | | | | | 2.303*** (0.130) |
| Observations | 14243 | 14243 | 14243 | 14243 | 14243 | 14243 |
| AIC | 85593.4779 | 54865.9120 | 55006.8616 | 54700.9022 | 55008.8616 | 54528.6162 |
| BIC | 85759.8863 | 55039.8845 | 55180.8341 | 54874.8747 | 55190.3981 | 54710.1527 |
| Log lik. | -42774.7 | -27410.0 | -27480.4 | -27327.5 | -27480.4 | -27240.3 |
| Dispersion | 6.67597 | constant | 1.998817 | | | |

Source, German Socioeconomic Panel (1984-1995)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$ **Table 4: Heterogeneous NB2 for Doctor Visit for Males and Females**

| | Het_NB2_Male | Het_NB2_Female |
|--------------|----------------------|----------------------|
| Docvis | | |
| Public | 0.0940 (0.0652) | 0.105* (0.0599) |
| Addon | 0.0427 (0.0927) | 0.0347 (0.0754) |
| Constant | 2.977*** (0.318) | 2.874*** (0.272) |
| Lnalpha | | |
| Public | 0.0188 (0.0985) | 0.0190 (0.102) |
| Addon | -0.397*** (0.153) | -0.495*** (0.142) |
| Constant | -0.839* (0.452) | -1.217*** (0.453) |
| Observations | 14243 | 13083 |
| AIC | 54493.3321 | 60278.6270 |
| BIC | 54826.1490 | 60607.7061 |
| Log lik. | -27202.7 | -30095.3 |

Source, German Socioeconomic Panel (1984-1995)

4.3 Zero-inflated models (Pooled data)

Following Greene (2008), pooled data is used for the purpose of selecting the best of available zero-inflated models. Since for the zero inflated models it is necessary to specify the inflation function, all the explanatory variables are covariates in this function. For the purpose of comparing the models we focus on the estimated coefficients for public insurance and add-on variables in the final reports.

Table 5 provides estimation results related to zero-inflated models for males. Based on the positive and statistically significant values of Vuong statistics (test for non-nested models), there is a strong evidence to prefer the zero-inflated models to their corresponding standard models. The AIC and BIC criteria are significantly lower for all the zero-inflated models when compared to their standard counterparts. This supports the idea that zero-inflated models are more suitable for describing the data than their standard models. Also, add-on contributes to the over-dispersion in the data as it is significant in the zero-inflated function. The Zero-Inflated Negative Binomial (ZINB) Waring model has the lowest AIC and BIC values followed by ZINB2 Famoye and ZINB2. The statistically significant estimated parameters related to dispersion in ZINB Waring and ZINB2 indicate that zero-inflated Poisson is not a good choice for these data and its significant coefficient for add-on is not reliable. The estimated parameters for add-on are positive for all of the models but statistically insignificant. Moreover, the public insurance coefficient is positive and statistically significant in zero-Inflated Poisson (ZIP) and ZINB2.

Table 6 provides the results of zero-inflated models for females. The results for females are similar to those for males. In the case of males, the coefficient for public insurance is statistically positive for ZINB2 model.

Table 5: Zero-Inflated Models for Males

| | ZIP | ZINB2 | ZINBFamoy | ZINBWaring |
|-----------------|-----------------------|----------------------|----------------------|----------------------|
| Docvis | | | | |
| Public | 0.0794*** (0.0247) | 0.0971* (0.0571) | 0.0899 (0.0560) | 0.0565 (0.0623) |
| addon | -0.0839* (0.0430) | -0.0388 (0.0962) | -0.0694 (0.0933) | -0.00574 (0.101) |
| Constant | 2.502*** (0.108) | 2.567*** (0.263) | -5.078 (154.3) | 2.598*** (0.291) |
| inflate | | | | |
| public | -0.0226 (0.0755) | 0.0342 (0.162) | 0.00727 (0.124) | -0.00330 (0.148) |
| addon | -0.423*** (0.157) | -0.651 (0.446) | -0.590* (0.316) | -0.637* (0.387) |
| Constant | -3.718*** (0.371) | -6.989*** (0.935) | -5.710*** (0.670) | -4.933*** (0.772) |
| Inalpha | | 0.154*** (0.0303) | | |
| Inphim1 | | | 6.560 (154.4) | |
| Intheta | | | 7.652 (154.3) | |
| Inrhom2 | | | | 0.866*** (0.0550) |
| Constant | | | | 0.897*** (0.103) |
| Observations | 14243 | 14243 | 14243 | 14243 |
| AIC | 70905.8533 | 54536.9885 | 54383.8199 | 54168.3164 |
| BIC | 71238.6702 | 54877.3694 | 54731.7649 | 54516.2613 |
| Log lik. | -35408.9 | -27223.5 | -27145.9 | -27038.2 |
| Vuong_statistic | 31.546871*** | 11.554577*** | 14.015931*** | 21.082627*** |

Standard errors in parentheses

Source, German Socioeconomic Panel (1984-1995)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Zero-Inflated Models for Females

| | ZIP | ZINB2 | ZINBFamoy | ZINBWaring |
|-----------------|----------------------|------------------------|----------------------|----------------------|
| docvis | | | | |
| public | 0.112*** (0.0217) | 0.0788* (0.0466) | 0.0674 (0.0459) | 0.0595 (0.0495) |
| addon | -0.0652* (0.0337) | -0.0146 (0.0768) | -0.0333 (0.0758) | 0.0649 (0.0753) |
| Constant | 2.307*** (0.0922) | 2.586*** (0.222) | -5.259 (134.6) | 2.663*** (0.228) |
| inflate | | | | |
| public | -0.0857 (0.0855) | -0.207 (0.194) | -0.149 (0.145) | -0.122 (0.165) |
| addon | -0.427** (0.176) | -0.913 (0.887) | -0.733 (0.476) | -0.459 (0.416) |
| Constant | -4.329*** (0.417) | -8.360*** (1.218) | -6.726*** (0.829) | -5.556*** (0.883) |
| Inalpha | | -0.0723*** (0.0258) | | |
| Inphim1 | | | 6.579 (134.7) | |
| Intheta | | | 7.879 (134.6) | |
| Inrhom2 | | | | 1.091*** (0.0464) |
| Ink | | | | 1.142*** (0.0868) |
| Observations | 13083 | 13083 | 13083 | 13083 |
| AIC | 79784.3595 | 60296.0349 | 60130.3157 | 59863.0979 |
| BIC | 80113.4386 | 60632.5930 | 60474.3529 | 60207.1351 |
| Log lik. | -39848.2 | -30103.0 | -30019.2 | -29885.5 |
| Vuong_statistic | 31.52*** | 8.80*** | 12.18*** | 12.67*** |

Standard errors in parentheses

Source, German Socioeconomic Panel (1984-1995)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

5. Model Evaluation

5.1. Distribution comparisons

In this section, the predictions (distributions) of Poisson and NB2 families are compared with the distribution from the actual data⁵. Figure 1 and Figure 2 compare the predicted frequency of the number of doctor visits with their actual frequencies. Both figures show that Poisson and zero-inflated Poisson models underestimate the zeros and overestimate the ones. Figure 3 compares the distributions results

⁵ Since the predicted values might not be integers, we convert them to an integer.

from the Poisson and zero-inflated Poisson models. We see that zero-inflated Poisson increases the estimated frequency of zeros by almost 40%, which is a substantial improvement in terms of prediction. We observe also some improvements in the reduction of the estimated 1 and 2 numbers of doctor visits. Regarding four and more visits, both models are nearly the same.

Figures 4 to 6 compare the results for NB2 and zero-inflated NB2. The improvement in the number of zeros using zero-inflated NB2 is almost 20% when compared with the standard NB2. The other results are similar to the ones obtained using Poisson family. The reduction in improvement of zero-inflated NB2 when compared to the improvement obtained from zero-inflated Poisson is due to the over-dispersion power of standard NB2 against standard Poisson model that was investigated in the previous sections. This can be seen in tables A4 to A7 of the appendix A. For example, standard Poisson distribution predicts correctly 838 zeros while standard NB2 predicts correctly 1170 zeros. With zero-inflated Poisson and zero-inflated NB2 these values increase to 1441 and 1446 respectively. Additionally, the zero-inflated models provide some improvements on the estimation of other numbers of doctor visits.

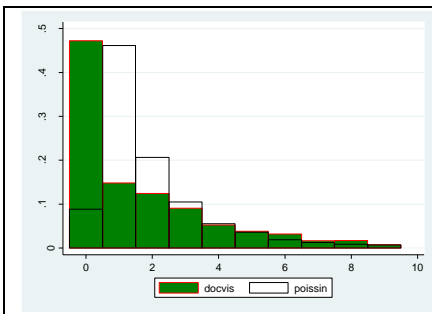


Figure1: Doctor visits (DV) and predicted DV (by Poisson)

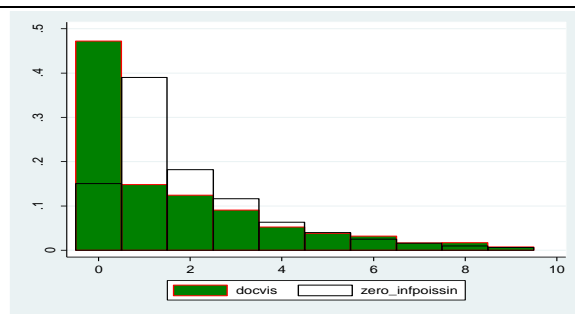


Figure2: DV and predicted DV (by ZPoisson)

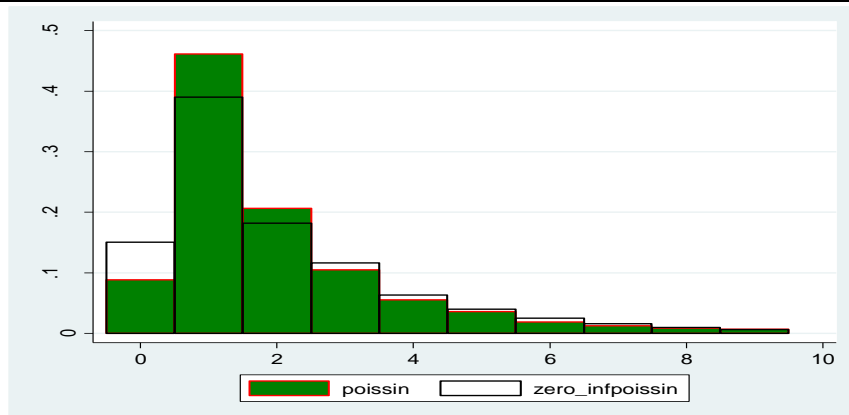


Figure3: Predicted DV by Poisson and ZPoisson

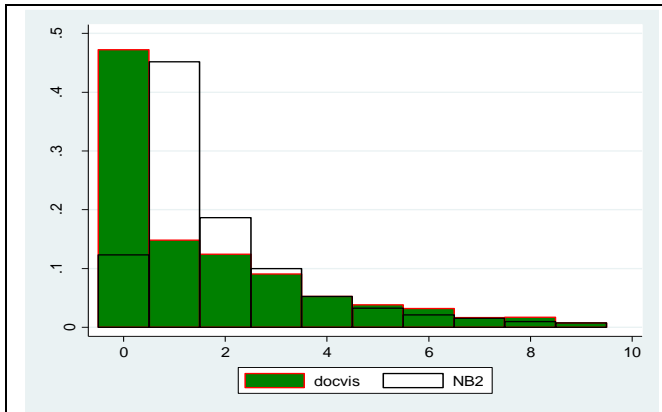


Figure4: DV and predicted DV (by NB2)

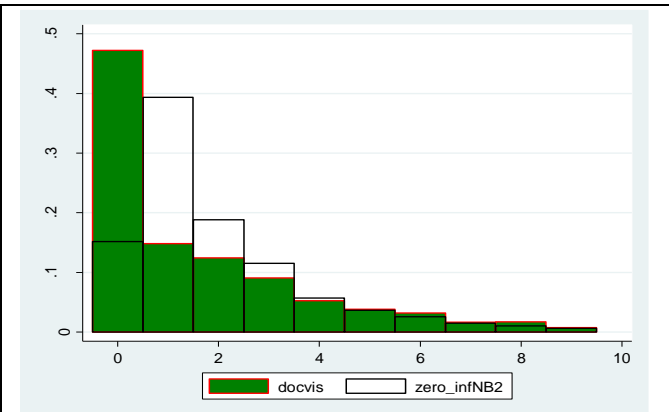


Figure5: DV and predicted DV (by ZNB2)

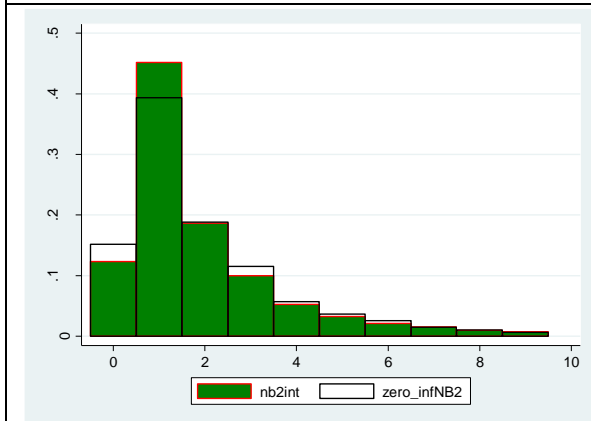


Figure6: Predicted DV by NB2 and ZNB2

5.2 Predicted versus realizations comparisons

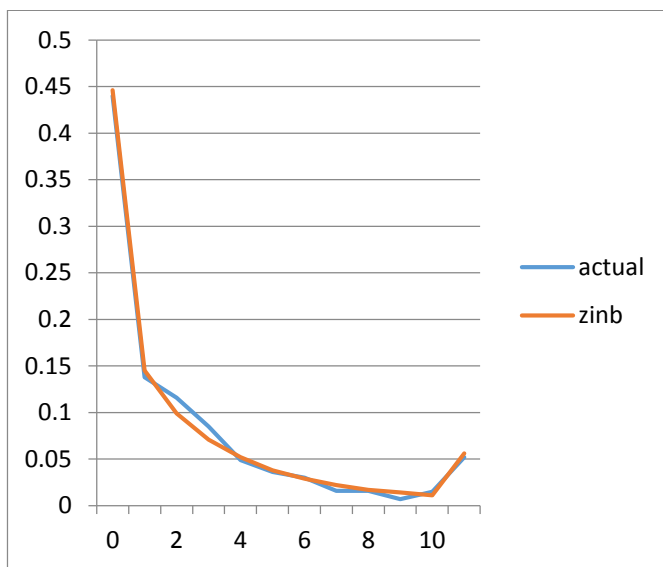
Table 7 compares four models based on the maximum differences and mean absolute differences between predicted and actual counts. The results show that Poisson performs worst at predicting the 0s, and NB2 and ZIP perform worst at predicting the 1s, while ZINB is worst at predicting the 2s. However, the maximum difference and mean of absolute differences are much lower for ZINB which means this model is the best one in terms of overall prediction. The Pearson statistic equals 193.429 for this model (the lowest of all models), which confirms it is the best model in terms of prediction (see table A8 in the appendix A for more details).

Figure 7 presents the density comparison between actual and predicted probabilities. Again, we see that ZINB is superior to ZIP in predicting actual probabilities. Further, Figure 8 plots the residuals from the tested models. Small residuals indicate a good fit, so the models with lines closest to zero should be considered as the suitable ones. We can see that the residuals line for ZINB is very close to zero when compared with the line of residuals for all the other models, confirming the results of all previous findings.

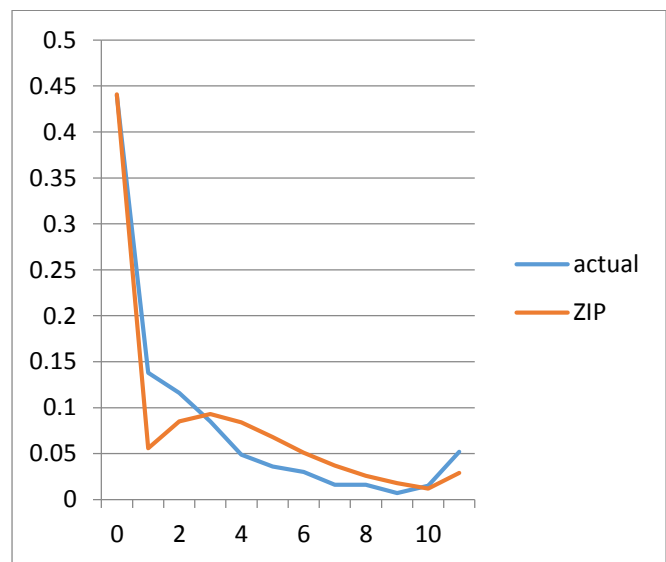
Table 7: Comparing of Mean observed and predicted count

| Model | Maximum Difference | At Value | Mean Diff |
|-------|--------------------|----------|------------|
| PRM | 0.273 | 0 | 0.054 |
| NBRM | -0.042 | 1 | 0.009 |
| ZIP | 0.082 | 2 | 0.023 |
| ZINB | 0.017 | 2 | 0.006 |

Figure 7: Density comparison between actual and predicted probabilities



Actual probabilities versus Zero Inflated Negative Binomial



Actual probabilities versus Zero Inflated Poisson

Figure 8: Residual plots of PRM(Poisson Random Effect Model, NBRM (NB Random Effect Model), ZIP(Zero Inflated Poisson Model), ZINB(Zero Inflated NB Model)

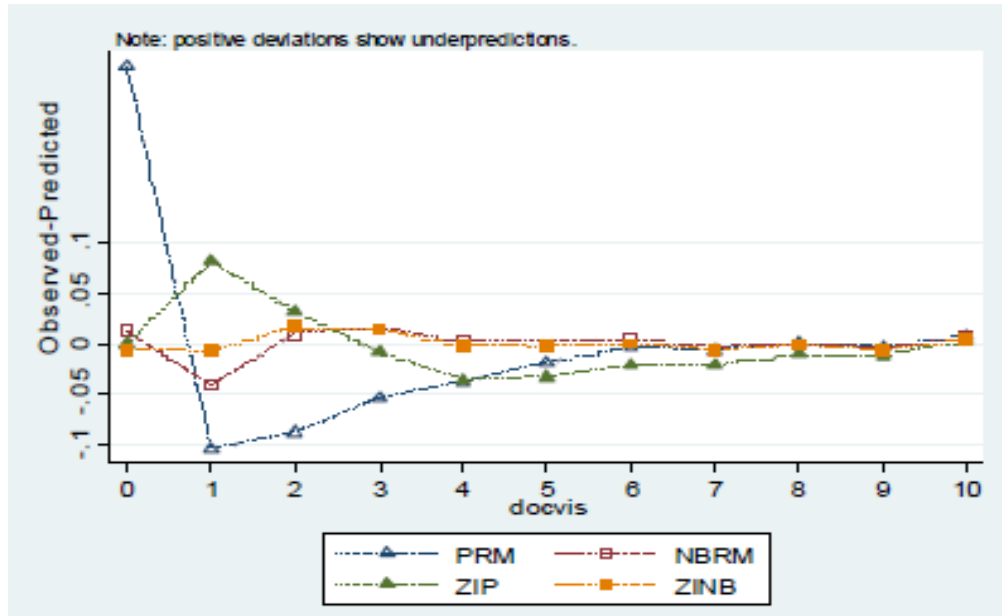


Table 8 provides tests for choosing the best model in terms of fit statistics such as AIC and BIC as well as Vuong statistic. The results also indicate that ZINB is the best model among the models under consideration.

Table 8: Tests of fit statistics

| PRM | BIC=85759.8 | AIC=85593.5 | Prefer | Over | Evidence |
|-------------|--------------------|--------------------|---------------|-------------|-----------------|
| Vs NBRM | BIC=55180.8 | Diff=30579.1 | NBRM | PRM | Very strong |
| | AIC=55006.8 | Diff=30586.6 | NBRM | PRM | |
| | LRX2=30588.6 | Prob=0.000 | NBRM | PRM | P=0.000 |
| Vs ZIP | BIC=71238.7 | Diff=14521.2 | ZIP | PRM | Very strong |
| | AIC=70905.8 | Diff=14687.6 | ZIP | PRM | |
| | Vuong=31.54 | Prob=0.000 | ZIP | PRM | P=0.000 |
| Vs ZINB | BIC=54877.4 | Diff=30882.5 | ZINB | PRM | Very strong |
| | AIC=54536.9 | Diff=31056.5 | ZINB | PRM | |
| NBRM | BIC=55180.8 | AIC=55006.8 | Prefer | Over | Evidence |
| Vs ZIP | BIC=71238.7 | Diff= - 16057.8 | NBRM | ZIP | Very strong |
| | AIC=70905.8 | Diff= - 15898.9 | NBRM | ZIP | |
| Vs ZINB | BIC=54877.4 | Diff=303.4 | ZINB | NBRM | Very strong |
| | AIC=54536.9 | Diff=469.8 | ZINB | NBRM | |
| | Vuong=11.55 | Prob=0.000 | ZINB | NBRM | P=0.000 |
| ZIP | BIC=71238.7 | AIC=70905.8 | Prefer | Over | Evidence |
| Vs ZINB | BIC=54877.4 | Diff=16361.3 | ZINB | ZIP | Very strong |
| | AIC=54536.9 | Diff=16368.8 | ZINB | ZIP | |
| | LRX2=16370.8 | Prob=0.000 | ZINB | ZIP | P=0.000 |

6. Robustness Checks

6.1 Robustness results on pooled sample

The results of previous sections show that zero-inflated NB2 could be considered a suitable model for both female and male subsamples. The model predicts that public insurance has a positive and statistically significant coefficient for both subsamples.

Checking if the ranking of models is preserved in a pooled sample (males and females) will complete our robustness checks. Consequently, six different models for the whole sample are estimated: Poisson with Gaussian random effect, Poisson with Gamma random effect, NB2, NB Waring, zero-inflated NB2 as well as zero-inflated NB Waring. A dummy variable with value of 1 for females and value of 0 for males is added to the explanatory variables. If its coefficient is statistically different from zero, it confirms that previous results based on subsamples still hold.

Table 9 reports the estimation results for the six models. In all of them the estimated coefficients for females are positive and statistically different from zero. This means that on average, *ceteris paribus*; females will demand more visits to doctors than males. This confirms the results obtained for the two separate samples. Between the random effect models, the one with Gamma distribution for the unobserved heterogeneity performs best (both AIC and BIC are predicting the same result). However, all the pooled data models are preferred to random-effect models. Further, the Young statistic used to compare non-nested models is positive and statistically significant for the zero-inflated models, meaning that zero-inflated models provide better predictions than their standard counterparts. Furthermore, based on AIC and BIC, the zero-inflated Waring model is preferred to the zero-inflated NB2. Finally, all the models estimate a positive coefficient for public insurance and are statistically significant except for the Gaussian random effect model.

Table 9: Full sample results

| | Gaussina RE | Gamma RE | NB2 | NBW | ZINB2 | ZINB Waring |
|-----------------|-----------------|------------------|-----------------|-----------------|-----------------|-----------------|
| <hr/> | | | | | | |
| docvis | | | | | | |
| female | 0.377*** | 0.304*** | 0.354*** | 0.364*** | 0.183*** | 0.215*** |
| | (0.0291) | (0.0244) | (0.0279) | (0.0234) | (0.0194) | (0.0207) |
| public | 0.0886 | 0.0896*** | 0.1000** | 0.0746* | 0.0930** | 0.0654* |
| | (0.0557) | (0.0264) | (0.0458) | (0.0393) | (0.0361) | (0.0389) |
| addon | -0.0299 | -0.0327 | 0.0497 | 0.148*** | -0.0193 | 0.0340 |
| | (0.0665) | (0.0347) | (0.0613) | (0.0553) | (0.0597) | (0.0606) |
| <hr/> | | | | | | |
| Insig2u | -0.111*** | | | | | |
| | (0.0277) | | | | | |
| lnalpha | | -0.136*** | 0.370*** | | 0.0243 | |
| | | (0.0205) | (0.0189) | | (0.0195) | |
| lnrhom2 | | | | 0.842*** | | 0.984*** |
| | | | | (0.0652) | | (0.0349) |
| lnk | | | | 2.280*** | | 1.043*** |
| | | | | (0.0923) | | (0.0665) |
| <hr/> | | | | | | |
| inflate | | | | | | |
| female | | | | | -1.216*** | -0.830*** |
| public | | | | | -0.0469 | -0.0477 |
| | | | | | (0.120) | (0.107) |
| addon | | | | | -0.682** | -0.562** |
| | | | | | (0.341) | (0.269) |
| <hr/> | | | | | | |
| Observations | 27326 | 27326 | 27326 | 27326 | 27326 | 27326 |
| AIC | 136878.2159 | 136666.2365 | 115861.5909 | 114914.2459 | 114881.7180 | 114051.5364 |
| BIC | 137075.3902 | 136863.4108 | 116058.7651 | 115119.6357 | 115267.8509 | 114445.8849 |
| Log lik. | -68415.1 | -68309.1 | -57906.8 | -57432.1 | -57393.9 | -56977.8 |
| vuong_statistic | | | | | 15.94*** | 39.64*** |

Standard errors in parentheses

Source, German Socioeconomic Panel (1984-1995)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

6.2 Accounting for correlation between doctor visits and hospital visits using the Bivariate Negative Binomial Model⁶

A bivariate NB model is estimated for both doctor visits and hospital visits to account for potential correlation between the two events. As Riphahn *et al* (2003) explain, doctor visits and hospital visits are positively correlated. However, this correlation should be identified and tested. In the males sample we find a correlation between doctor visits and hospital visits of 0.1477. Using only non-zero values, this correlation reduces to 0.1138. Cameron and Trividi (2013) show how to construct a statistic for testing the independency between two counts specific regressions (for doctor visits and hospital visits). The calculated test statistic for our sample are 9.47, 1.05, 0.05, 0.94, respectively for $(j, k) = (1,1), (1,2), (2,1), (2,2)$. Only the first value is statistically different from zero (with p-value equal to 0.002). This shows that independency can only be rejected suggesting some evidence of weak dependency between the two count variables, which is in contrast to what Riphahn *et al* (2003) expected. Motivated by the first test value, a bivariate NB2 model is estimated for males using pooled panel data (see Table 10). Here the parameter α can capture both overdispersion as well as correlation between unobserved heterogeneity⁷.

For females, the correlation between doctor visits and hospital visits for all data realizations (including the zeros) is 0.125 and when we look at only the positive values we get a correlation of 0.079. The independency test statistic for females are 38.56, 0.087, 4.81, 0.225 corresponding to $(j, k) = (1,1), (1,2), (2,1), (2,2)$, respectively. Based on the results, the first and second values of the test statistic are significant at 0.05 level.

Motivated by the first two test values, a bivariate NB2 is estimated for female. The results are presented in Table 10. We see that the α parameters are significant in both male and female bivariate models. This result might confirm the previous results that NB2 is a better distribution for explaining real data, rather than bivariate Poisson distribution. Here we find that the estimated public insurance parameters for doctor visits are positive and statistically significant for both males and females while for hospital visits are negative and insignificant. Although estimated add-on parameters are positive for all males and females in two equations, they are statistically insignificant.

6 Codes for Bivariate NB2 model are found in Cameron, C., and Trividi, P. (2013) (see page 336)

⁷ For the estimating Bivariate NB2 by ML, initial values we find by estimating non-linear seemingly unrelated regression (NLSUR) and assuming initial value for α equal to 2. For first stage, correlation between doctor visits and hospital visits for males and females are estimated as 0.125 and 0.078, respectively.

Table 10: Bivariate Negative Binomial 2 for Doctor Visits and Hospital Visits

| | Males | | Females | |
|--------------|----------------------------------|--------------------------|-----------------------------|--------------------------|
| | Doctor visit | Hospital visit | Doctor visit | Hospital visit |
| age | -0.0384*** (0.0126) | -0.00765 (0.0519) | -0.0315*** (0.00956) | -0.0945** (0.0376) |
| age2 | 0.531*** (0.145) | 0.0924 (0.603) | 0.375*** (0.108) | 0.917** (0.437) |
| hsat | -0.238*** (0.00652) | -0.253*** (0.0304) | -0.207*** (0.00564) | -0.223*** (0.0240) |
| handdum | -0.0181 (0.0507) | -0.128 (0.149) | 0.115** (0.0528) | 0.126 (0.122) |
| handper | 0.00654*** (0.000941) | 0.00703*** (0.00261) | 0.00433*** (0.000867) | 0.00914*** (0.00248) |
| married | 0.0700 (0.0446) | -0.0271 (0.145) | 0.0241 (0.0317) | -0.00770 (0.114) |
| educ | -0.0269*** (0.00719) | -0.0883*** (0.0300) | 0.00762 (0.00700) | -0.0416 (0.0346) |
| hhninc | - 0.0000205** (0.00000918) | 0.0000293 (0.0000257) | -0.0000163* (0.00000845) | 0.0000749 (0.0000220) |
| hhkids | -0.0890** (0.0396) | 0.0623 (0.130) | -0.122*** (0.0310) | 0.0134 (0.108) |
| self | -0.214*** (0.0658) | -0.0280 (0.229) | -0.244*** (0.0731) | -0.366* (0.201) |
| beamt | 0.0825 (0.0602) | -0.0923 (0.264) | -0.0218 (0.108) | -0.108 (0.264) |
| bluec | 0.0356 (0.0377) | 0.0556 (0.158) | -0.0401 (0.0414) | -0.414*** (0.141) |
| working | -0.0200 (0.0512) | -0.0232 (0.195) | 0.0322 (0.0303) | 0.0391 (0.0900) |
| public | 0.0958* (0.0551) | -0.173 (0.235) | 0.105* (0.0545) | -0.144 (0.312) |
| addon | 0.0605 (0.0863) | 0.550 (0.351) | 0.0311 (0.0648) | 0.0501 (0.164) |
| d85 | 0.0902 (0.0598) | 0.462* (0.251) | -0.0118 (0.0484) | 0.0131 (0.161) |
| d86 | 0.223*** (0.0609) | -0.0480 (0.182) | 0.102** (0.0472) | 0.204 (0.173) |
| d87 | 0.121* (0.0642) | 0.145 (0.187) | -0.0542 (0.0627) | 0.0872 (0.205) |
| d88 | 0.0595 (0.0571) | -0.00514 (0.169) | -0.173*** (0.0464) | 0.370** (0.186) |
| d91 | -0.00918 (0.0551) | -0.139 (0.172) | -0.129*** (0.0449) | 0.371** (0.168) |
| d94 | 0.242*** (0.0568) | 0.0794 (0.182) | 0.220*** (0.0476) | -0.0102 (0.169) |
| Constant | 3.658*** (0.273) | 1.107 | 3.176*** (0.231) | 1.926 (1.184) |
| Alpha | 1.698*** (0.0367) | | 1.169*** (0.0242) | |
| Observations | 14243 | | 13083 | |
| Log Like | -33090.1 | | -36174.5 | |

Standard errors in parentheses
Source, German Socioeconomic Panel (1984-1995)

7. Conclusion

In summary, the high share of zeros for a dependent variable in a count data regression model can severely increase the over-dispersion issue and lead to unreliable estimators. We show that the German Socioeconomic Panel (1984-1995) used by Riphahn et al (2003) for the demand of doctor visits suffers severely from over-dispersion issue and their estimation based on standard distributions might not be reliable. Results based on standard distributions are close to Riphahn et al (2003) and, overall, there is not enough evidence for moral hazard and adverse selection except for Waring NB2, which presents a positive effect from adverse selection on the number of doctor visits. However, this result might also not be reliable due to the over-dispersion that resulted in the high share of zeros in the data. The paper shows that, for this data, within the class of random effect models, the model with a Gamma distribution for unobserved heterogeneity is more suitable than the one assuming Gaussian distribution for unobserved heterogeneity.

To select between non-nested models a Vuong test (1989) is employed. The test rejects the standard distributions in the favour of their corresponding zero-inflated distributions. This means that over-dispersion due to the high share of zeros in the data cannot be explained by any complex and/or flexible mixture of Poisson distributions such as Negative Binomial 1 and 2, Generalized Poisson, Negative Binomial Famoy, Generalized Negative Binomial Waring models. All of these are inferior to the zero-inflated distributions models. Between zero-inflated distributions, ZINB Waring model has the lowest AIC and BIC values followed by ZINB2 Famoye and ZINB2. However, when ranking the predicted probabilities, ZINB2 model produces the closest probabilities to the actual probabilities. A pooled (male-female) sample estimation provides the same results as those obtained from subsample estimations.

All zero-inflated distribution models predict the coefficient of add-on as positive but statistically insignificant for both male and female subsamples (same predictions as Riphahn *et al* (2003) in their bivariate model). The coefficient of public insurance is found positive, but only for ZINB2 is it statistically significant for both genders. A similar result is obtained from an estimated bivariate NB. This is in contrast with the results by Riphahn *et al* (2003), who did not find any significant coefficient for public insurance in their bivariate model. This shows that moral hazard plays a positive and significant role for visiting more doctors. The results provide significant evidence of how considering the over-dispersion nature of the data in the estimation process can provide more precise estimations and reveal a better understanding about health demand components.

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Appendix A: Additional Tables

Table A1

| Variables | Description | Males ^a | | Females ^a | |
|----------------|--|--------------------|---------|----------------------|---------|
| <i>docvis</i> | Number of doctor visits in last three months | 2.626 | (5.21) | 3.791 | (6.11) |
| <i>Hos</i> | Number of hospital visit last year | 0.128 | (0.93) | 0.150 | (0.83) |
| <i>Age</i> | Age | 42.653 | (11.27) | 44.467 | (11.32) |
| <i>Hsat</i> | Health satisfaction code 0 (low)-10 (high) | 6.924 | (2.25) | 6.634 | (2.33) |
| <i>handdum</i> | Person is handicapped (0/1) | 0.227 | (0.42) | 0.200 | (0.40) |
| <i>handper</i> | Percentage degree of handicap | 8.134 | (20.33) | 5.791 | (17.96) |
| <i>married</i> | Person is married (0/1) | 0.765 | (0.42) | 0.752 | (0.43) |
| <i>educ</i> | Years of schooling | 11.729 | (2.44) | 10.876 | (2.11) |
| <i>hhninc</i> | Monthly household net income ($\times 10^{-3}$) | 3.591 | (1.74) | 3.445 | (1.80) |
| <i>hhkids</i> | Children below age 16 in household (0/1) | 0.413 | (0.49) | (0.392) | (0.49) |
| <i>Self</i> | Person is self-employed (0/1) | 0.086 | (0.28) | 0.037 | (0.19) |
| <i>beamt</i> | Person is civil servant (0/1) | 0.118 | (0.32) | 0.028 | (0.16) |
| <i>bluec</i> | Person is blue collar worker (0/1) | 0.340 | (0.47) | 0.139 | (0.35) |
| <i>working</i> | Person is employed (0/1) | 0.850 | (0.36) | 0.488 | (0.50) |
| <i>public</i> | Person is insured in public health insurance (0/1) | 0.861 | (0.35) | 0.913 | (0.28) |
| <i>addon</i> | Person is insured in add-on insurance (0/1) | 0.018 | (0.13) | 0.020 | (0.14) |
| <i>d85</i> | Year=1985 (0/1) | | | | |
| <i>d86</i> | Year=1986 (0/1) | | | | |
| <i>d87</i> | Year=1987 (0/1) | | | | |
| <i>d88</i> | Year=1988 (0/1) | | | | |
| <i>d91</i> | Year=1991 (0/1) | | | | |
| <i>d94</i> | Year=1994 (0/1) | | | | |
| <i>N</i> | Number of observations | 14,243 | | 13,083 | |

^a mean, standard deviation in parentheses

Source: German Socioeconomic Panel (1984–1995).

Table A2: Standard Distributions for Doctor Visit for Males (complete table)

| | poisson | NB1 | NB2 | Gpoisson | NBFamoy | GNBWaring |
|--------------|----------------------------|---------------------------|----------------------------|----------------------------|----------------------------|---------------------------|
| docvis | | | | | | |
| age | -0.0239 (0.0164) | -0.0477*** (0.0114) | -0.0398*** (0.0153) | -0.0496*** (0.0114) | -0.0398*** (0.0153) | -0.0533*** (0.0120) |
| age2 | 0.369** (0.184) | 0.634*** (0.129) | 0.547*** (0.176) | 0.659*** (0.130) | 0.547*** (0.176) | 0.706*** (0.137) |
| hsat | -0.225*** (0.00767) | -0.189*** (0.00585) | -0.239*** (0.00745) | -0.188*** (0.00587) | -0.239*** (0.00745) | -0.203*** (0.00657) |
| handdum | 0.0690 (0.0537) | 0.0229 (0.0378) | -0.0209 (0.0503) | 0.0183 (0.0373) | -0.0209 (0.0503) | 0.0111 (0.0397) |
| handper | 0.00286** (0.00121) | 0.00414*** (0.000848) | 0.00661*** (0.00116) | 0.00430** (0.000835) | 0.00661*** (0.00116) | 0.00505*** (0.000917) |
| married | 0.0583 (0.0606) | 0.130*** (0.0408) | 0.0658 (0.0535) | 0.135*** (0.0409) | 0.0658 (0.0535) | 0.139*** (0.0432) |
| educ | -0.0235*** (0.00873) | -0.00955 (0.00672) | -0.0262** (0.00910) | -0.00833 (0.00688) | -0.0262*** (0.00910) | -0.00971 (0.00725) |
| hhninc | -0.0000222* (0.0000121) | -0.0000788 (0.0000853) | -0.0000192* (0.0000105) | -0.00000746 (0.0000868) | -0.0000192* (0.0000105) | -0.0000835 (0.0000911) |
| hhkids | -0.0760 (0.0518) | -0.0743** (0.0341) | -0.0844* (0.0470) | -0.0766** (0.0343) | -0.0844* (0.0470) | -0.0792** (0.0362) |
| self | -0.211** (0.0847) | -0.244*** (0.0616) | -0.218*** (0.0784) | -0.253*** (0.0628) | -0.218*** (0.0784) | -0.265*** (0.0656) |
| beamt | 0.0914 (0.0809) | 0.0278 (0.0623) | 0.0841 (0.0766) | 0.0273 (0.0630) | 0.0841 (0.0766) | 0.0254 (0.0664) |
| bluec | 0.0178 (0.0486) | -0.00948 (0.0374) | 0.0371 (0.0458) | -0.0116 (0.0379) | 0.0371 (0.0458) | -0.00956 (0.0398) |
| working | -0.0554 (0.0668) | 0.0126 (0.0465) | -0.0155 (0.0596) | 0.0175 (0.0465) | -0.0155 (0.0596) | 0.0172 (0.0490) |
| public | 0.100 (0.0702) | 0.0607 (0.0539) | 0.0934 (0.0635) | 0.0595 (0.0549) | 0.0934 (0.0635) | 0.0578 (0.0577) |
| addon | 0.0666 (0.102) | 0.139* (0.0777) | 0.0551 (0.0948) | 0.144* (0.0791) | 0.0551 (0.0948) | 0.154* (0.0844) |
| d85 | 0.0769 (0.0563) | 0.0615* (0.0359) | 0.106* (0.0546) | 0.0611* (0.0358) | 0.106* (0.0546) | 0.0669* (0.0378) |
| d86 | 0.215*** (0.0597) | 0.156*** (0.0365) | 0.226*** (0.0581) | 0.155*** (0.0365) | 0.226*** (0.0581) | 0.163*** (0.0386) |
| d87 | 0.113 (0.0690) | 0.0967** (0.0439) | 0.123** (0.0613) | 0.0983** (0.0433) | 0.123** (0.0613) | 0.104** (0.0458) |
| d88 | 0.0530 (0.0558) | 0.111*** (0.0360) | 0.0670 (0.0544) | 0.110*** (0.0361) | 0.0670 (0.0544) | 0.115*** (0.0379) |
| d91 | -0.00397 (0.0609) | 0.145*** (0.0373) | -0.00366 (0.0531) | 0.152*** (0.0374) | -0.00366 (0.0531) | 0.151*** (0.0393) |
| d94 | 0.247*** (0.0613) | 0.268*** (0.0407) | 0.244*** (0.0548) | 0.278*** (0.0409) | 0.244*** (0.0548) | 0.289*** (0.0430) |
| Constant | 2.771*** (0.336) | 2.776*** (0.254) | 3.149*** (0.329) | 2.780*** (0.258) | 3.710*** (0.330) | 2.929*** (0.273) |
| Lndelta, | | 1.581*** (0.0365) | | | | |
| Lnalpha | | | 0.561*** (0.0270) | | | |
| atanhdelta | | | | 0.726*** (0.0115) | | |
| Lnphim1 | | | | | -17.76*** (3.253) | |
| lntheta | | | | | -0.561*** (0.0270) | |
| lnrhom2 | | | | | | 0.783*** (0.0981) |
| lnk | | | | | | 2.303*** (0.130) |
| Observations | 14243 | 14243 | 14243 | 14243 | 14243 | 14243 |
| AIC | 85593.4779 | 54865.9120 | 55006.8616 | 54700.9022 | 55008.8616 | 54528.6162 |
| BIC | 85759.8863 | 55039.8845 | 55180.8341 | 54874.8747 | 55190.3981 | 54710.1527 |
| Dispersion | 6.67597 | | constant | 1.998817 | | |
| Log lik. | -42774.7 | -27410.0 | -27480.4 | -27327.5 | -27480.4 | -27240.3 |

Standard errors in parentheses

Source, German Socioeconomic Panel (1984-1995)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A3: Standard Distributions for Doctor Visit for Females (complete)

| | poisson | NB1 | NB2 | Gpoisson | NBFamoy | GNBWaring |
|--------------|------------------------------|----------------------------|-----------------------------|----------------------------|-----------------------------|----------------------------|
| docvis | | | | | | |
| age | -0.0132 (0.0121) | -0.0322*** (0.00943) | -0.0312*** (0.0115) | -0.0347*** (0.00947) | -0.0321*** (0.0116) | -0.0400*** (0.0112) |
| age2 | 0.179 (0.138) | 0.396*** (0.107) | 0.373*** (0.131) | 0.425*** (0.107) | 0.382*** (0.132) | 0.479*** (0.127) |
| hsat | -0.203*** (0.00641) | -0.171*** (0.00507) | -0.208*** (0.00631) | -0.170*** (0.00506) | -0.208*** (0.00636) | -0.218*** (0.00609) |
| handdum | 0.138** (0.0565) | 0.106** (0.0450) | 0.113** (0.0485) | 0.102** (0.0434) | 0.111** (0.0487) | 0.119** (0.0480) |
| handper | 0.00241** (0.00108) | 0.00254*** (0.000867) | 0.00436*** (0.00106) | 0.00254*** (0.000846) | 0.00457*** (0.00110) | 0.00418*** (0.000998) |
| married | 0.0272 (0.0408) | 0.0440 (0.0322) | 0.0282 (0.0385) | 0.0455 (0.0323) | 0.0284 (0.0386) | 0.0366 (0.0377) |
| educ | 0.0147 (0.00933) | 0.0138* (0.00724) | 0.00773 (0.00894) | 0.0136* (0.00728) | 0.00740 (0.00898) | 0.0121 (0.00873) |
| hhninc | -0.0000206** (0.00000948) | -0.0000111 (0.00000740) | -0.0000162* (0.00000951) | -0.0000103 (0.00000746) | -0.0000161* (0.00000955) | -0.0000128 (0.00000916) |
| hhkids | -0.134*** (0.0416) | -0.108*** (0.0311) | -0.124*** (0.0376) | -0.108*** (0.0311) | -0.124*** (0.0375) | -0.122*** (0.0367) |
| self | -0.218** (0.0978) | -0.223*** (0.0705) | -0.242*** (0.0875) | -0.229*** (0.0707) | -0.244*** (0.0872) | -0.280*** (0.0849) |
| beamt | -0.0711 (0.117) | -0.00922 (0.0848) | -0.0198 (0.128) | -0.00859 (0.0859) | -0.0183 (0.129) | -0.0499 (0.107) |
| bluec | -0.0354 (0.0555) | -0.0718* (0.0392) | -0.0401 (0.0497) | -0.0772** (0.0392) | -0.0406 (0.0495) | -0.0730 (0.0471) |
| working | 0.0149 (0.0392) | 0.0247 (0.0294) | 0.0305 (0.0354) | 0.0264 (0.0295) | 0.0313 (0.0354) | 0.0363 (0.0347) |
| public | 0.131** (0.0599) | 0.0790 (0.0489) | 0.0953 (0.0639) | 0.0715 (0.0499) | 0.0935 (0.0643) | 0.0787 (0.0598) |
| addon | 0.0207 (0.0888) | 0.126* (0.0682) | 0.0309 (0.0769) | 0.138** (0.0687) | 0.0312 (0.0769) | 0.111 (0.0794) |
| d85 | -0.0362 (0.0473) | -0.0326 (0.0319) | -0.0127 (0.0449) | -0.0303 (0.0318) | -0.0119 (0.0450) | -0.0218 (0.0386) |
| d86 | 0.0941** (0.0449) | 0.0837** (0.0329) | 0.102** (0.0430) | 0.0836** (0.0328) | 0.102** (0.0433) | 0.114*** (0.0383) |
| d87 | -0.0843 (0.0642) | -0.0750 (0.0485) | -0.0531 (0.0566) | -0.0690 (0.0471) | -0.0515 (0.0569) | -0.0701 (0.0529) |
| d88 | -0.180*** (0.0448) | -0.0677** (0.0315) | -0.176*** (0.0439) | -0.0670** (0.0315) | -0.176*** (0.0441) | -0.145*** (0.0384) |
| d91 | -0.154*** (0.0456) | 0.0108 (0.0326) | -0.138*** (0.0441) | 0.0202 (0.0327) | -0.138*** (0.0442) | -0.0688* (0.0402) |
| d94 | 0.197*** (0.0481) | 0.186*** (0.0370) | 0.221*** (0.0464) | 0.191*** (0.0371) | 0.222*** (0.0466) | 0.252*** (0.0433) |
| Constant | 2.547*** (0.282) | 2.731*** (0.224) | 3.024*** (0.273) | 2.777*** (0.227) | 3.184*** (0.276) | 3.190*** (0.267) |
| Indelta | | 1.549*** (0.0349) | | | | |
| lnalpha | | | 0.188*** (0.0259) | | | |
| atanhdelta | | | | 0.711*** (0.0108) | | |
| lnphim1 | | | | | -4.580*** (0.762) | |
| lntheta | | | | | -0.133*** (0.0443) | |
| lnrhom2 | | | | | | 1.014*** (0.113) |
| lnk | | | | | | 0.283*** (0.0764) |
| Observations | 13083 | 13083 | 13083 | 13083 | 13083 | 13083 |
| AIC | 91844.4596 | 60731.5683 | 60570.6248 | 60521.2975 | 60569.0256 | 60307.5709 |
| BIC | 92008.9991 | 60903.5869 | 60742.6434 | 60693.3160 | 60748.5232 | 60487.0686 |
| Log lik. | -45900.2 | -30342.8 | -30262.3 | -30237.6 | -30260.5 | -30129.8 |
| Dispersion | 6.689348 | | 1.487322 | | | |

Standard errors in parentheses

Source, German Socioeconomic Panel (1984-1995)

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table A4: Predicted and original numbers of doctor visits based on Standard Poisson model

| docvis | poissonint | | | | | | |
|--------|------------|-------|-------|-------|-----|-----|-----|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 838 | 3,564 | 1,089 | 437 | 165 | 77 | 35 |
| 1 | 183 | 976 | 424 | 205 | 85 | 36 | 19 |
| 2 | 104 | 721 | 421 | 189 | 87 | 63 | 22 |
| 3 | 56 | 441 | 289 | 158 | 103 | 74 | 25 |
| 4 | 19 | 225 | 162 | 111 | 60 | 38 | 23 |
| 5 | 12 | 135 | 124 | 72 | 56 | 42 | 18 |
| 6 | 7 | 109 | 92 | 64 | 57 | 26 | 23 |
| 7 | 3 | 60 | 49 | 36 | 22 | 15 | 5 |
| 8 | 3 | 52 | 64 | 30 | 26 | 14 | 13 |
| 9 | 1 | 22 | 15 | 16 | 16 | 7 | 5 |
| Total | 1,226 | 6,305 | 2,729 | 1,318 | 677 | 392 | 188 |

Table A5: Predicted and original numbers of doctor visits based on zero-inflated Poisson model

| docvis | zpoissonint | | | | | | | | | | Total |
|--------|-------------|-------|-------|-------|-----|-----|-----|-----|----|----|--------|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| 0 | 1,441 | 2,931 | 981 | 502 | 205 | 96 | 44 | 31 | 17 | 15 | 6,263 |
| 1 | 294 | 845 | 378 | 239 | 100 | 48 | 24 | 17 | 8 | 7 | 1,960 |
| 2 | 167 | 643 | 380 | 211 | 103 | 62 | 39 | 16 | 12 | 10 | 1,643 |
| 3 | 94 | 390 | 256 | 175 | 114 | 79 | 39 | 20 | 9 | 11 | 1,187 |
| 4 | 34 | 206 | 143 | 112 | 65 | 46 | 29 | 22 | 11 | 10 | 678 |
| 5 | 21 | 122 | 112 | 68 | 66 | 43 | 29 | 16 | 13 | 3 | 493 |
| 6 | 17 | 92 | 75 | 79 | 54 | 29 | 29 | 17 | 6 | 10 | 408 |
| 7 | 10 | 50 | 38 | 46 | 23 | 14 | 9 | 8 | 7 | 4 | 209 |
| 8 | 8 | 44 | 54 | 39 | 25 | 18 | 12 | 8 | 3 | 1 | 212 |
| 9 | 3 | 18 | 10 | 18 | 16 | 9 | 8 | 3 | 7 | 0 | 92 |
| Total | 2,089 | 5,341 | 2,427 | 1,489 | 771 | 444 | 262 | 158 | 93 | 71 | 13,145 |

Table A6: Predicted and original numbers of doctor visits based on Standard NB2 model

| docvis | nb2int | | | | | | |
|--------|--------|-------|-------|-------|-----|-----|-----|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1,170 | 3,360 | 971 | 406 | 172 | 63 | 42 |
| 1 | 242 | 954 | 386 | 199 | 73 | 39 | 23 |
| 2 | 137 | 735 | 369 | 187 | 72 | 58 | 33 |
| 3 | 77 | 442 | 266 | 147 | 91 | 69 | 35 |
| 4 | 27 | 235 | 145 | 97 | 56 | 40 | 21 |
| 5 | 18 | 139 | 116 | 60 | 60 | 38 | 23 |
| 6 | 10 | 113 | 80 | 66 | 45 | 24 | 23 |
| 7 | 7 | 59 | 44 | 38 | 21 | 12 | 4 |
| 8 | 5 | 55 | 54 | 33 | 23 | 15 | 9 |
| 9 | 1 | 22 | 13 | 16 | 15 | 7 | 4 |
| Total | 1,694 | 6,114 | 2,444 | 1,249 | 628 | 365 | 217 |

Table A7: Predicted and original numbers of doctor visits based on zero-inflated NB2 model

| docvis | znb2int | | | | | | |
|--------|---------|-------|-------|-------|-----|-----|-----|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0 | 1,446 | 2,932 | 1,020 | 478 | 184 | 94 | 41 |
| 1 | 289 | 860 | 385 | 238 | 85 | 40 | 27 |
| 2 | 171 | 647 | 387 | 216 | 85 | 55 | 41 |
| 3 | 97 | 388 | 266 | 177 | 102 | 68 | 44 |
| 4 | 35 | 210 | 145 | 109 | 64 | 39 | 27 |
| 5 | 20 | 126 | 110 | 81 | 53 | 40 | 28 |
| 6 | 18 | 95 | 82 | 72 | 45 | 36 | 23 |
| 7 | 8 | 54 | 39 | 42 | 23 | 13 | 7 |
| 8 | 8 | 47 | 52 | 41 | 24 | 16 | 10 |
| 9 | 3 | 18 | 9 | 19 | 12 | 14 | 5 |
| Total | 2,095 | 5,377 | 2,495 | 1,473 | 677 | 415 | 253 |

Table A8: Comparison of predicted and actual probabilities

NB2: Predicted and actual probabilities

| Count | Actual | Predicted | Diff | Pearson |
|-------|--------|-----------|-------|---------|
| 0 | 0.440 | 0.428 | 0.012 | 4.992 |
| 1 | 0.138 | 0.180 | 0.042 | 136.630 |
| 2 | 0.116 | 0.106 | 0.011 | 15.150 |
| 3 | 0.085 | 0.069 | 0.016 | 52.273 |
| 4 | 0.049 | 0.048 | 0.001 | 0.617 |
| 5 | 0.036 | 0.034 | 0.001 | 0.800 |
| 6 | 0.030 | 0.025 | 0.005 | 13.029 |
| 7 | 0.016 | 0.019 | 0.004 | 9.693 |
| 8 | 0.016 | 0.015 | 0.001 | 0.983 |
| 9 | 0.007 | 0.012 | 0.005 | 25.303 |
| 10 | 0.015 | 0.009 | 0.006 | 55.167 |
| Sum | 0.948 | 0.944 | 0.103 | 314.635 |

ZINB: Predicted and actual probabilities

| Count | Actual | Predicted | Diff | Pearson |
|-------|--------|-----------|-------|---------|
| 0 | 0.440 | 0.446 | 0.006 | 1.139 |
| 1 | 0.138 | 0.145 | 0.007 | 5.034 |
| 2 | 0.116 | 0.099 | 0.017 | 40.468 |
| 3 | 0.085 | 0.071 | 0.014 | 40.148 |
| 4 | 0.049 | 0.052 | 0.003 | 1.765 |
| 5 | 0.036 | 0.038 | 0.003 | 2.878 |
| 6 | 0.030 | 0.029 | 0.001 | 0.591 |
| 7 | 0.016 | 0.022 | 0.007 | 28.703 |
| 8 | 0.016 | 0.017 | 0.001 | 1.759 |
| 9 | 0.007 | 0.014 | 0.006 | 44.089 |
| 10 | 0.015 | 0.011 | 0.005 | 26.856 |
| Sum | 0.948 | 0.945 | 0.070 | 193.429 |

ZIP: Predicted and actual probabilities

| Count | Actual | Predicted | Diff | Pearson |
|-------|--------|-----------|-------|----------|
| 0 | 0.440 | 0.441 | 0.000 | 0.000 |
| 1 | 0.138 | 0.056 | 0.082 | 1724.211 |
| 2 | 0.116 | 0.085 | 0.032 | 169.678 |
| 3 | 0.085 | 0.093 | 0.008 | 10.143 |
| 4 | 0.049 | 0.084 | 0.035 | 211.840 |
| 5 | 0.036 | 0.068 | 0.033 | 222.485 |
| 6 | 0.030 | 0.051 | 0.021 | 124.089 |
| 7 | 0.016 | 0.037 | 0.021 | 175.386 |
| 8 | 0.016 | 0.026 | 0.010 | 55.840 |
| 9 | 0.007 | 0.018 | 0.011 | 94.030 |
| 10 | 0.015 | 0.012 | 0.003 | 9.078 |
| Sum | 0.948 | 0.971 | 0.256 | 2796.778 |

Appendix B: Model Specifications

A Poisson model, $f(y_i, \mu) = \frac{\mu_i^{y_i} e^{-\mu_i}}{y_i!}$; $\mu_i > 0, y_i = 0, 1, 2, \dots$, with $E(Y_i) = V(Y_i) = \mu_i = \exp(X_i\beta) \rightarrow \ln(\mu_i) = X_i\beta$, cannot deal with the over-dispersion due to the equality of mean and variance. To make it more flexible the model can be augmented with other distributions. This is done by relating its mean to an individual unobserved effect (u_i), such that: $\ln(\text{mean}_i) = \ln(\mu_i) + \ln(u_i)$. It follows that the conditional distribution of y_i with respect to X_i and u_i can be written as: $f(y_i|X_i, u_i) = \frac{e^{-(\mu_i u_i)} (\mu_i u_i)^{y_i}}{y_i!}$, and the unconditional distribution can be written as $f(y_i|X_i) = \int_0^\infty \frac{e^{-(\mu_i u_i)} (\mu_i u_i)^{y_i}}{y_i!} g(u_i) du_i$. Different extensions of Poisson can be obtained depending on how the distribution for u_i is specified. Table 2 presents a list of distributions with their variances, distributions that are used in the proposed analysis as robustness checks of different model specifications.

Table B1: The list of standard distributions

| Distribution | Variance |
|--|---|
| Poisson | μ (where $\ln \mu = X\beta$) |
| Negative Binomial 1 (NB1) | $\mu + \alpha\mu$ (where α is the dispersion parameter) |
| Negative Binomial 2 (NB2) | $\mu + \alpha\mu^2$ |
| Generalized Negative Binomial (NB-P) | $\mu + \alpha\mu^p$ |
| Heterogeneous Negative Binomial (NB-H) | $\mu + \alpha_i\mu^2$ (where $\alpha_i = z_i\gamma$) |
| Generalized Negative Binomial (Famoy) | $\theta\mu(1 - \phi\mu)(1 - \phi\mu)^{-3}$ |
| Waring Negative Binomial (NBW) | $\mu + \mu \left(\frac{k+1}{\rho-2} \right) + \mu^2 \left\{ \frac{k+\rho-1}{k(\rho-2)} \right\}$ |
| Generalized Poisson, (GP) | $\frac{1}{(1-\delta)^2} \mu$ |

Assuming a specific distribution for u_i it can be shown that it produces a new distribution with higher variance than Poisson model. For instance, in the case u_i has a Gamma distribution, with mean 1 and variance $\alpha = \frac{1}{\theta}$, the mixture of Poisson-Gamma is called Negative Binomial 2 (NB2) which has mean μ_i and conditional variance $\mu_i + \alpha\mu_i^2$. The Negative Binomial 1 (NB1) model is obtained by assuming $\theta_i = \theta\mu_i$. In NB1 and NB2, α refers to heterogeneity parameter. In addition, when $\alpha = 0$ both distributions will be identical to Poisson.

Besides the NB1 and NB2, we consider the power negative binomial (NB-P) distribution, which has an additional parameter, p , see Greene (2008) to estimate. For $p = 1$ and $p = 2$, this distribution collapses to NB1 and NB2, respectively.

In NB2, the dispersion parameter, α , is constant. The NB2 can be generalized to a heterogeneous NB2 model that allows the dispersion parameter to vary across observations. In this case: $\alpha = \exp(z_i\gamma)$. This gives more flexibility to the dispersion parameter to capture most of the variation in the data.

Another generalized version of the NB models is introduced by Famoye (1995), which has the following mean and variances:

$$E(Y) = \theta\mu(1 - \phi\mu)^{-1} \quad V(Y) = \theta\mu(1 - \mu)(1 - \phi\mu)^{-3} ,$$

where θ is an unknown to be estimated and parameter $\phi > 1$. With these parameters, the variance is bigger than NB model. When $\phi \rightarrow 1$, the variance of this distribution approaches to that of the NB.

Further, we consider univariate generalized Warning distribution or the beta Negative Binomial model, which is another extension of Negative Binomial model that was introduced by Irwin (1968). This model has more flexibility for specifying the unobservable heterogeneity. In other words, as Hilbe (2014) explains “unobserved heterogeneity can be separated from the internal factors of each individual’s characteristics and external factors (covariates) that may affect the variability of data”(see Hilbe 2011). This distribution is specified under the following assumptions (Irwin 1968):

$$Y|x, \mu_x, v \sim \text{Poisson}(\lambda_x)$$

$$\lambda_x|v \sim \text{Gamma}(a_x, u), \quad a_x > 0$$

$$v \sim \text{Beta}(\rho, k), \quad \rho > 2, k > 0$$

$$E(Y) = \frac{a_x k}{\rho - 1} = \mu = \ln(X\beta) \quad V(Y) = \mu + \mu \left(\frac{k+1}{\rho-2} \right) + \mu^2 \left\{ \frac{k+\rho-1}{k(\rho-2)} \right\}$$

If $\left(\frac{k+1}{\rho-2} \right) \rightarrow 0$ and $\left\{ \frac{k+\rho-1}{k(\rho-2)} \right\} \rightarrow 0$, or equivalently if $k, \rho \rightarrow \infty$, the variance of the distribution will converge to $V(Y) = \mu$ like Poisson distribution. Also, If $\left(\frac{k+1}{\rho-2} \right) \rightarrow 0$ and $\left\{ \frac{k+\rho-1}{k(\rho-2)} \right\} \rightarrow \alpha$, or equivalently if $k \rightarrow \frac{1}{\alpha}, \rho \rightarrow \infty$, the variance of the distribution will converge to the one for NB2 ($V(Y) = \mu + \alpha\mu^2$).

Moreover, we also employ a Generalized Poisson (GP) model with density $f(y_i; \theta_i, \delta) = \frac{\theta_i(\theta_i + \delta y_i)^{y_i-1} e^{-\theta_i - \delta y_i}}{y_i!}$, $0 \leq \delta < 1$; $E(Y_i) = \mu_i = \frac{\theta_i}{1-\delta}$ $V(Y_i) = \frac{\theta_i}{(1-\delta)^3} = \frac{1}{(1-\delta)^2} E(Y_i)$.

When $\delta = 0$, the GP distribution reduces to the usual Poisson distribution with parameter θ_i . In addition, when $\delta > 0$ the model assumes over-dispersion, and when it is less than zero, the model assumes under-dispersion.

2.1 Zero-inflated count models

Another source of over-dispersion is the presence of a large number of zeros in the data. In other words, when there are more zeros than the expected number of zero under a specified distribution, the data may be over-dispersed. Zero-inflated Poisson (ZIP) model and zero-inflated negative binomial (ZINB) models adjust for excessive zeros in the response. Recently, Hilbe and *et al* (2014) discuss different versions of zero-inflated models.

Zero-inflated models consider two distinct sources of zero outcomes. One source is generated from individuals who do not enter the counting process, the other from those who do enter the counting process but result in zero outcomes. As an example, suppose that we have data related to the number of visits made to the office of patient's physician. However, we might have individuals in our data that they do not have any physician. Then the probability of observing a zero outcome equals the probability that an individual is in the group that does not have physician ($B(0)$) plus the probability that the individual is in the other group multiplies the probability he does not see the physician $\{1 - B(0)\}Pr(0)$. $Pr(0)$ is the counting process that produces zero; see Hilbe and Greene (2008). Thus, the model has two parts:

- 1) The probability of a zero outcome for the system is given by⁸:

$$Pr(y = 0) = B(0) + \{1 - B(0)\}Pr(0)$$

- 2) And the probability of a nonzero count is⁹:

$$Pr(y = k; k > 0) = \{1 - B(0)\}Pr(k)$$

$Pr(k)$ is the counting process that the outcome is k . Logistic and Probit models are used for B and Poisson or Negative Binomial or other distributions are used for $Pr(k)$. This mixture provides more power for the model to explain the sources of over-dispersion in the data.

Table B2 presents different zero-inflated distributions that are used in the next sections for the purpose of estimation and comparison.

⁸ Stata gives this probability using the command: predict f0, pr(0)

⁹ Stata gives this probability using the command: predict fk, pr(k)

TableB2: Zero-inflated distributions

Zero-inflated Poisson (ZIP)

Zero-inflated Negative Binomial 1 (ZINB1)

Zero-inflated Negative Binomial 2 (ZINB2)

Zero-inflated Generalized NB (ZINB-P)

Zero-inflated Poisson Inverse Gaussian, (ZIPIG)

Zero-inflated Generalized Poisson, (ZIGP)

Zero-Inflated 3-parameter Waring NB (ZINBW)

Zero-inflated 3-parameter Famoye NB (ZINBF)
