Buffer Joint Ventures

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Abstract

While strategic alliances and joint ventures have become important organizational forms promising a variety of efficiency benefits for the economy, a body of research has been building showing that alliances between competitors can have significant anticompetitive consequences. This paper explores a particular kind of arrangement, here called a “buffer joint venture”, in which parent firms create an entity selling products located between their own locations in product or geographic space. Depending upon the governance structure of the joint venture and the timing of price-setting by the joint venture and its parents, the buffer joint venture may reduce competition between the parents leading to higher prices and profits and lower social welfare. The presence of such a joint venture can also affect the incentives for, and the effects of, collusion by the parents.
I. Introduction

As has been recognized in a long literature, strategic alliances and joint ventures have become increasingly common in a wide variety of industries, perhaps most famously among airlines, pharmaceutical companies and automobile manufacturers. While definitions vary, in our use of the terms, strategic alliances involve the ongoing cooperation of firms to provide new or improved products or services (on the output side), or to improve production/distribution methods (on the input side). Airline alliances, for example, can include the sharing of aircraft and interlining agreements to transfer passengers efficiently. Joint ventures can be interpreted as the efforts of two or more independent firms to create a new, shared entity, to provide a new product or service. As such they represent a particular kind of strategic alliance. Standing somewhere intermediate in the spectrum between standard market-mediated relations between firms and complete integration, such arrangements promise a number of benefits to participants including the sharing of fixed costs, the combining of complementary talents and a way to facilitate the sharing of intellectual property.

An extensive body of scholarly work on strategic alliances has developed in the management and economics literatures. Management scholars have, for example, measured the increased frequency of these types of arrangements in a variety of industries, undertaken detailed case studies and created guidance on the factors that contribute generally to successful collaborations. The economics literature has provided a number of models to explain why firms might want to cooperate these ways, and explored the potential effects on rivals and consumers.

As has been shown in a number of papers, when the firms cooperating in an alliance are otherwise competitors in some markets, these kinds of arrangements can pose risks to competition. For this reason a number of competition authorities have been paying increasing attention to alliances, for example by subjecting them to a kind of review similar to that designed

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2 Much of this literature is discussed below in section II.
for mergers. Some competition authorities have gone so far as to issue guidelines on competitor collaborations.

This paper explores the potential for anticompetitive effects from a type of alliance not previously studied for this purpose. This type of alliance involves the creation by parent firms of a joint venture that lies between them in product or geographic space. Through their joint control of this new entity, the parents place a buffer between themselves, softening competition and potentially raising prices. While the new product can undoubtedly add value for some consumers, the overall effect on consumer surplus and total welfare can be negative.

The best examples of these kinds of alliances may be found in the automotive sector where firms might jointly produce products that combine key characteristics of the parents. Also relevant would be alliances in the pharmaceutical industry in which, for example, brand name pharmaceutical companies partner with generic producers to manufacture branded generic drugs.

We study two ways through which this “buffer JV” can affect competition. First, we examine the effects on parents’ prices and welfare of the creation of the JV when the parents’ prices are still set non-cooperatively. Second, we explore the effect of introducing a JV on incentives to collude and on the effects of that collusion.

We show that the buffer JV can indeed raise the prices and profits of parents even if the parent brand prices are set non-cooperatively. The magnitudes of the effects depend on the governance structure of the JV and whether or not the JV is assigned the role of price leader in the market. We also find that the presence of a buffer JV can make collusion more or less profitable, depending on the magnitude of fixed costs.

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3 This has become common for airline alliances, for example. On recent developments related to the granting of antitrust immunity for airline alliances in the U.S. see, Moss (2018). See also the list of alliances reviewed for antitrust immunity by the U.S. Department of Transportation at: https://cms.dot.gov/sites/dot.gov/files/docs/mission/office-policy/aviation-policy/9906/170104-all-immunized-alliances-010318.pdf (updated January 2018).


5 For example, the famous General Motors and Toyota joint venture, begun in the 1980s, which combined Toyota’s talents at quality small-car production with GM’s marketing and distribution abilities (and familiarity with supply chains and labor relations in the United States). The partners produced new versions of the Toyota Corolla (the GM version called the Chevrolet Nova). On this alliance, see Kwoka (1989). A more recent example is the joint venture between Ford and the Chinese electric vehicle manufacturer Zotyne to create new models of electric automobiles. See, for example Reuters (2017).

6 An example of this might be the partnership between the U.S. company Merck & Co. and India’s Sun Pharmaceutical Industries to produce and market “branded generic” versions of Merck products. See, Karmali (2011). See also a discussion on branded generics at Singer (2010).
We view this work as supportive of the concerns shared by many competition authorities regarding the possible harmful effects on competition that can arise when competitors cooperate even on what might not be viewed as competitive variables. We demonstrate a new set of conditions under which such harm is possible.

Section II reviews some of the key literature related to the effects of joint ventures on competition and describes the place of this paper in that literature. Section III then lays out the basic static model of competition on which we build, and describes the pre-JV benchmark against which our joint venture outcomes will be compared. The following sections, examine equilibria with joint ventures in cases where all price-setting is simultaneous (Section IV) and where the JV is made a price-leader (Section V). Section VI explores questions related to the profitability and effects of collusion with and without a JV operating between its parents. Section VII then describes some implications of these results for competition policy and offers our conclusions.

II. Related Literature and Contributions of this Paper
As noted above, there is now an extensive literature measuring the prevalence of, examining the motives for, and evaluating the effects of strategic alliances and joint ventures. Here we very briefly review some of the key work done on the potential for anticompetitive effects through such arrangements.

Past research has identified a number of ways that alliances can impact competition negatively. One possible mechanism relates to the aligning of incentives that comes from cross ownership or joint/common ownership. Even when partners make their pricing decisions completely independently, if they have minority ownership shares in their rivals – a not uncommon element in alliance agreements – their incentives to compete aggressively can be muted. Early work in this area included important papers by Reynolds and Snapp (1986) and Bresnahan and Salop (1986) whose ideas were expanded upon in O’Brien and Salop (2000). The potential effects of joint or common ownership of multiple firms operating in a single

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7 See also the more recent work by Shelegia and Spiegel (2012). Brito et al. (2014) showed how one could build a merger simulation model to predict the unilateral price effects of partial horizontal acquisitions. For some of the antitrust implications see Shapiro and Willig (1990).
market has become a hot topic more recently, fueled by the prominence of a few large private equity investors in certain industries, such as airlines and banks.\(^8\)

A second mechanism leading to the potential for competitive harm involves the use of input-joint ventures to control the amount of output produced in a market downstream from the joint venture, as described, for example by Carlton and Salop (1996), Kabiraj and Chaudhuri (2001) and Chen and Ross (2003). A large related literature on research joint ventures has developed here as well, including papers by Katz (1986), Grossman and Shapiro (1986), D’Aspremont et al. (1988), Jorde and Teece (1990) and Kamien et al. (1992).\(^9\)

A third, and related, mechanism involves the uses of alliance-like structures to induce the sharing of monopoly to deter full-scale entry that would lead to a more competitive duopoly. Examples here include Chen and Ross (2000) as well as Gallini’s (1984) examination of licensing.

A fourth mechanism involves the use of alliances to facilitate collusion. This is a common concern expressed, often in informal ways, for example by: (i) explaining that alliance meetings could be used as a cover for cartel meetings; or (ii) by reference to how cooperating with respect to some variables – even if by itself efficient – can lead firms to move toward cooperation with respect to other variables, perhaps because a build-up of trust or a culture of cooperation. More formal treatments can be found in Gilo et al. (2006) and Cooper and Ross (2009).

Our contribution here describes a mechanism that, while somewhat related to the first and fourth mechanisms above, is novel in important ways. The creation of a new buffer joint venture located between the parents’ locations in product or geographic space means that each parent will now compete more directly with the JV than with the other parent. If the parents can cooperate on the setting of the JV’s prices (and other strategic variables), they can use it to soften the competition between themselves. Even if the joint venture is run as an independent profit-maximizing entity its decisions will be influenced by the interests of its parents and competition


\(^{9}\) There is also a literature on patent pooling which can have elements of these first two potentially anticompetitive effects – horizontal “overlapping ownership” (see Gallini, 2014) when intellectual property (IP) holders participate in the development of a rival product standard, and vertical relationships in which the holders of important IP can work together to limit downstream production. See also Lerner and Tirole (2004) and Lerner et al (2007). We thank Nancy Gallini for drawing our attention to the parallels between some kinds of joint ventures and patent pooling agreements.
may be softened as a result, as has been described in the common ownership literature. While sharing the idea of common ownership with previous work, the key difference here is that the shared-ownership is of an entirely new entity. While one might expect that the creation of a new producer would enhance competition to some degree at least, here we will see that this is not necessarily the case – in fact the joint venture can harm competition.

We also show that the presence of a JV located between its parents has implications for the profitability of collusion. If fixed costs of establishing the JV are low enough, collusion will be more profitable with a JV than without it – and, regardless of the level of fixed costs, more harmful to consumers. This is another mechanism, then, through which the presence of a JV could support collusion by parents – one that does not derive from the creation of trust or a culture of cooperation, nor does it come from influencing equilibrium selection in a game with multiple equilibria.10

Finally, we explore the relative profitability of two collusive strategies parents may consider. We will have seen that JVs – particularly more cooperative ones – can raise two parents’ total profits. But simple collusion by the parents on the pricing of their two products can also raise profits. We show here that establishing a cooperative JV without any explicit collusion on parents’ own product prices can be as profitable, and in some cases possibly more profitable, as colluding on the two prices. Importantly, the JV helps generate these profits with a (presumably) substantially lower antitrust risk for the parents.

In its study of the introduction of a new brand into product space, our paper is also connected to the literature on filling the product space to deter entry, as famously introduced by Schmalensee (1978). While the motive for using the new product to deter entry does exist in our model, we will largely focus on the competition between the parents (and not on entry).11

III. Model and Benchmark

To illustrate the effects we want to highlight, we use the simplest possible model that gives us firms competing with differentiated products and allows them to place a jointly owned venture

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10 As discussed, for example, in Cooper and Ross (2009). Also, Morasch (2000) examines how parents may be able to use an input joint venture to support collusion. In a related way – but again without a new JV entity – Gilo et al (2006) study the influence of partial cross-ownership on the potential for tacit collusion.

11 This is to say that, if entry by a third independent firm were possible, the benefits to the parents of establishing a JV will be even greater than what we describe here.
(JV) between them in product or geographic space. That is, the JV will have some of the
attributes of each parent and will be more similar to the parents than the two parents are similar
to each other.

We do this with a model of competition along a Hotelling line with firms/brands located
in fixed positions at the two ends. The line is of unit length and buyers are uniformly distributed
along the line with density $M$. Firms have constant marginal costs but incur a fixed, sunk cost of
$F$ for each brand. We will further assume that the original fixed costs of the parents have already
been sunk; they will have no effect (and will be ignored) in the following analysis. The fixed
cost associated with establishing the JV will be important, however. Without any further loss of
generality, we set the marginal cost to zero.

The utility of a consumer from consuming a unit of good located at distance $d$ is $V - p - td$.
That is, there is a finite reservation value $V$ for goods of the exact type preferred by
customers, while the willingness to pay for more distant brands will fall linearly with that
distance according to the disutility (“transport cost”) parameter $t$. We assume that $V$ is
sufficiently large that the market is covered in all scenarios studied below. Specifically, we
assume $V \geq (11/6) t$.$^{12}$

Two firms, Firm 1 and Firm 2, serve this market. Initially, each firm produces and sells
one good. Let $A$ denote the good produced by Firm 1 and $B$ the good produced by Firm 2. As
noted, goods $A$ and $B$ are located at each end of the line, respectively.

The two firms have the option of setting up a joint venture (Firm J) to produce and sell a
third good, $C$. Let $x$ denote the location of good $C$ on the unit interval. The unit cost of
production of good $C$ is the same as that of goods $A$ and $B$. We assume that $x = 0.5$, that is,
Product $C$ is located at the center of the line.$^{13}$

$^{12}$ In the standard Hotelling model with two goods, the assumption $V \geq 1.5 t$ is sufficient to ensure that the market is
covered in equilibrium. Here we assume a higher threshold of $V$ in order to reduce the number of special cases
(related to narrow ranges in the value of $V$) we need to analyze. The analysis of cases under $V \geq \left(\frac{11}{6}\right) t$ is sufficient
to demonstrate our ideas about a buffer JV.

$^{13}$ For our purposes, we assume the parents place the JV in the middle of the product space though were we to let
them optimize the location choice other locations might be possible. Our assumption is mostly for convenience –
other locations will require non-symmetric JV arrangements. It could also be justified as being the location that
would be chosen if costs increased in a quadratic fashion with distance; if filling the middle served an additional
purpose of discouraging entry by an outside firm; or if firms were reluctant to hand over too much sensitive
information to their rivals for purpose of operating the JV (the middle location suggesting that they are making
roughly equal sensitive contributions.)
In this paper, we are interested in exploring the two firms’ incentives to offer product C through a joint venture. Specifically, Firm 1 and Firm 2 play the following two-stage game:

**Stage 1**: Firms 1 and 2 decide whether they want to form a joint venture (with equal ownership shares) to produce good C (located at $x = 0.5$).

**Stage 2**: The firms (all those in the market) choose the prices of their respective products.

Regarding stage 2, we consider both simultaneous price-setting and sequential price-setting (with the joint venture acting as the price-leader and the parent firms as price-followers). We think it is natural to consider a price-leadership version of the model for reasons of both motive and opportunity. As will become clear below, price-leadership by the joint venture is profitable for the parents. And it is not difficult to imagine that the communication between parents necessary to implement the JV could take this into account and lead parents to see the value in JV price-leadership. Put another way, the issue of JV price-leadership can be viewed as one of the design features contemplated by parent firms. The policy of following their JV’s lead can be seen to be a sort of facilitating practice on the part of parents, likely falling short of criminal cartel conduct.

We use $p_i$ and $Q_i$ to denote the price and quantity of good $i$ ($i = A, B, C$). The demand functions for the three goods can be shown to be:

$$Q_A = M \left( \frac{1}{4} - \frac{p_A - p_C}{2t} \right); \quad Q_B = M \left( \frac{1}{4} - \frac{p_B - p_C}{2t} \right); \quad Q_C = M \left( \frac{1}{2} - \frac{2p_C - p_A - p_B}{2t} \right). \quad (1)$$

The gross profit generated by product $i$ (i.e. profit before deducting the fixed cost in the case of good C) is then $\pi_i = p_iQ_i$. Let $\Pi_i$ denote firm $i$’s ($i = 1, 2$) gross profit. In the scenario where the two firms set up the joint venture, $\Pi_1 = \pi_A + \pi_C/2$ and $\Pi_2 = \pi_B + \pi_C/2$. Otherwise, $\Pi_1 = \pi_A$ and $\Pi_2 = \pi_B$.

**Governance and the Joint Venture’s Objective Function**

One of the key dimensions along which joint ventures can differ – one that is central to our analysis here – is the degree to which the JV is run independently of its parents. In our model, this is formalized in the objective function to be maximized by the JV as it sets its price. At one extreme, the JV could be run completely independently from its parents such that it
selects its price based solely on what maximizes JV profits. This independence might be the result of a free choice by the parents who believe it to be a better way to motivate and monitor appointed JV leadership. Alternatively, independence could be required by competition authorities worried about having competitors collaborate too closely. Below we will refer to such as case as involving an “independent JV”.

At the other extreme, the JV could be completely controlled by its parents such that its price is chosen to maximize its parents’ total profits (i.e. profits from their own products plus the JV’s profits). We call this a “cooperative JV”. Between these extremes will be cases in which JV puts some weight on its parents’ profits, but less weight than on its own profits.

We model this range of possibilities by assuming that the objective function of the JV takes the form \( \Pi_{JV} = \alpha(\pi_A + \pi_B) + \pi_C \), where \( \alpha \in [0, 1] \). In other words, the JV’s incentives will be to maximize the sum of its own profit and a share of its owners’ profits. When \( \alpha = 0 \) we have the case of an independent JV and when \( \alpha = 1 \) the case of a fully cooperative JV. We will refer to the case of \( 0 < \alpha < 1 \) as one of a “partially cooperative” JV.

In the analysis that follows we first consider the effects from introducing a new joint venture for a given, exogenous, value of \( \alpha \). We then also explore the case of an endogenous \( \alpha \), chosen by parents to maximize their total profits. While it might appear intuitive that parents would choose fully cooperation (i.e. \( \alpha = 1 \)), we will see that this is not always the case. Since each parent chooses its own price to maximize its profits (including a share of JV profits) even with a fully cooperative JV we are not in a fully collusive situation in the market.

**Benchmark: Two-Product Equilibrium**

To assess the effects on prices, profits and welfare measures of the introduction of a JV, we first need to establish those values for the pre-JV (“status quo”) comparator benchmark. This would also be the equilibrium of the two-stage game if fixed costs were so large as to make a JV unprofitable. That is, consider the equilibrium at status quo, where only good A and good B are sold. The demand functions for these two goods are:\(^{14}\)

\(^{14}\) As the techniques for solving these models are very familiar, we are relegating many of the derivations here to the Appendix.
\[ Q_i = M \left( \frac{1}{2} - \frac{p_i - p_j}{2t} \right), \quad (i, j = A, B, i \neq j). \quad (2) \]

In this standard, textbook, model it is straightforward to find the equilibrium prices of the two products: \( p_A^O = p_B^O = t \). The equilibrium profit of each firm (we ignore the sunk fixed costs for incumbents) is:

\[ \Pi_1^O = \Pi_2^O = \frac{Mt}{2}. \quad (3) \]

Recall that the utility of a consumer from consuming a unit of good located at distance \( d \) is \( V - p - td \). In this benchmark case where two products are sold at the same price \( p \), the aggregate consumer surplus is

\[ CS = M \left[ \int_0^{0.5} [V - p - t\theta] d\theta + \int_{0.5}^1 [V - p - t(1 - \theta)] d\theta \right] = M \left( V - \frac{t}{4} - p \right). \quad (4) \]

Given the equilibrium prices above, consumer surplus here will be:

\[ CS^O = M \left[ V - \frac{5}{4} t \right]. \quad (5) \]

Our measure of social welfare is the standard total surplus (consumer surplus plus firm profits) and we use these terms interchangeably. In this benchmark case, we sum profits from (3) and consumer surplus from (5) to obtain:

\[ TS^O = M \left( V - \frac{t}{4} \right). \quad (6) \]

**IV. Joint Venture with Simultaneous Price-Setting**

As described above the model has two versions in terms of whether the joint venture sets its prices at the same time as its parents or whether (as part of the JV agreement perhaps) the JV is a price leader. In this section, we consider the scenario in which the joint venture chooses its price at the same time as its parent firms, and we will analyze price leadership in the next section.

It is important to recognize that, in this section and the next, the parents will always be assumed to be setting their own product prices non-cooperatively (though internalizing effects on
their shares of JV profits).\footnote{Section VI considers the case in which parents set their product prices cooperatively.} This means each parent may find it profitable to undercut the price of the JV sufficiently to take over its whole market share. As the JV is in the center of the space between them, it is conceivable that a parent may wish to set a price so low that it steals all the JV’s market share and then finds itself competing on the margin with the other parent. As observed below, this indeed occurs if the price of the JV exceeds a certain threshold and this will have an effect on the equilibrium.

To analyze the scenario in which the joint venture chooses its price at the same time as its parent firms, we use (1) to write the objective function of each parent firm as:

\[
\Pi_i = \pi_j + \frac{\pi_c}{2} = p_j \left( \frac{1}{4} - \frac{p_j - p_c}{2t} \right) M + (p_c/2) \left( \frac{1}{2} - \frac{2p_c - p_j - p_k}{2t} \right) M \tag{7}
\]

where \((i, j, k) = (1, A, B)\) and \((2, B, A)\). Maximizing (7) for each parent firm, we find their best-response functions:

\[
p_A = \frac{t + 3p_c}{4}. \tag{8}
\]

\[
p_B = \frac{t + 3p_c}{4}. \tag{9}
\]

Eq. (8) and (9) prescribe the prices of the parent firms as continuous functions of the JV’s price \(p_c\). In fact, however, there will be a discontinuity in the demand functions for products A and B as functions of the price of C. If \(p_c\) is high enough, while small price cuts below the levels given in (8) and (9) remain unprofitable, larger price cuts that take over the JV’s whole market share will be profitable. As is shown in the appendix, (8) and (9) represent the parent firms’ best responses only if \(p_c \leq \frac{5}{3}t\). When \(p_c\) exceeds \(\frac{5}{3}t\), a parent firm will in fact find it profitable to deviate from (8)-(9) and cut its price by an amount large enough to take over all the JV’s sales. We will refer to such a response by a parent firm as a discrete or drastic price deviation.

With this in mind, we solve the JV’s optimization problem at stage 2:
\[
\max_{p_c} \Pi_{J\!V} = \alpha p_A \left( \frac{1}{4} - \frac{p_A - p_c}{2t} \right) M + \alpha p_B \left( \frac{1}{4} - \frac{p_B - p_c}{2t} \right) M \\
+ p_c \left( \frac{1}{2} - \frac{2p_c - p_A - p_B}{2t} \right) M, \quad (10)
\]
to find the JV’s best-response function:

\[
p_c = \frac{t + (1 + \alpha)p_A + (1 + \alpha)p_B}{4}. \quad (11)
\]

Solving the equation system of (8), (9) and (11), we derive the equilibrium prices at stage 2 (for a fixed \(\alpha\)):

\[
p_A = p_B = \frac{7t}{2(5 - 3\alpha)}, \quad p_c = \frac{(3 + \alpha)t}{(5 - 3\alpha)}. \quad (12)
\]

Recalling that (8) and (9) are applicable as long as \(p_c \leq \left(\frac{5}{3}\right)t\), we use (12) to find that the latter implies \(\alpha \leq 8/9\). In other words, (12) gives the equilibrium prices at stage 2 for \(\alpha \in [0, 8/9]\).

There are a number observations we can make based on (12) and comparisons with the results from the benchmark case. We will see that several of the qualitative effects on prices, quantities and profits will depend on whether or not \(\alpha\) is above or below \(1/2\).\(^{16}\)

First, from (12), it is straightforward to show that equilibrium prices in this range are such that \(\frac{\partial p_i}{\partial \alpha} > 0\). In words, a larger \(\alpha\) (i.e. more cooperative JV) leads to higher prices for all three goods.

Comparing price levels to those prior to the JV: \(p_i < p_i^O\) if \(\alpha < 1/2\) and \(p_i \geq p_i^O\) if \(\alpha \in [1/2, 8/9]\) where \(p_i^O = t\) is the equilibrium price in the two-good benchmark. That is, when the JV is more independent (\(\alpha < 1/2\)), introducing the JV is a net stimulus to price competition.\(^{17}\)

Since each parent only claims half of the profits of the JV, it does represent some level of new and closer competition. However, the incentive to cut prices on the parent’s own products is dulled by the fact of partial ownership of this new rival. With low values of \(\alpha\), the net effect is to

\[^{16}\text{The case in which } \alpha=1/2 \text{ corresponds to what O’Brien and Salop (2000, p. 583) referred to, in a slightly different context, as “proportional control” – where the JV (in our case) makes pricing decisions taking into account the profits of each shareholder in proportion to their ownership of the JV.}

\[^{17}\text{Though prices will not be as low as in the case of entry by an independent third firm, when price would fall to } t/2 \text{ for all three products. See the Appendix.}\]
depress price. When the JV is more cooperative, however, \((i.e. \alpha > 1/2)\), prices will rise relative to the pre-JV situation.

If \(\alpha > 8/9\), on the other hand, there is no pure-strategy equilibrium at stage 2. To illustrate this, consider the special case where \(\alpha = 1\). Setting \(\alpha = 1\) in (12), we find that \(p_A = p_B = \left(\frac{4}{7}\right)t\) and \(p_C = 2t\). Given such a high \(p_C\) and the price of the other parent firm, firm \(i\) will engage in a drastic price deviation to steal the entire market share from the JV. Since both parents have this incentive to drastically cut their prices, they bypass the JV on the product line and compete with each other directly, causing the prices of goods A and B to fall towards the equilibrium prices of two-good benchmark \((p_A^0 = p_B^0 = t)\). On the other hand, the JV will respond by lowering its price to \(p_C = \left(\frac{5}{3}\right)t\) in an attempt to capture a positive market share. Facing this lower \(p_C\), the parents will raise their prices as per their best-response functions (8) and (9). But given these prices of goods A and B, the JV’s best response is to raise \(p_C\) above \(\left(\frac{5}{3}\right)t\). As a result, only a mixed-strategy equilibrium is possible in this case.

Returning to the case \(\alpha \leq 8/9\), we substitute (12) into (1) to find the equilibrium quantities

\[
Q_A = Q_B = \frac{(4 - \alpha)M}{4(5 - 3\alpha)} , \quad Q_C = \frac{(6 - 5\alpha)M}{2(5 - 3\alpha)} .
\]  

(13)

Note that, in this model, the quantities translate easily into market shares: setting \(M = 1\) in (13) reveals the market shares of each product. Different values of \(\alpha\) will lead to different relevant market shares of the JV compared to its parents. Specifically: \(\partial Q_A / \partial \alpha = \partial Q_B / \partial \alpha > 0\) and \(\partial Q_C / \partial \alpha < 0\). In words, when the JV is more cooperative, it sells less and its parents sell more. When \(\alpha = ½\) the JV is exactly twice as large as each of its parents.

Using (12) and (13), we obtain the equilibrium profit of a parent firm in this case:

\[
\Pi_i - \frac{F}{2} = \frac{(64 - 25\alpha - 10\alpha^2)Mt}{8(5 - 3\alpha)^2} - \frac{F}{2} , \quad (i = 1,2) .
\]  

(14)

Not surprisingly, more cooperative JVs are more profitable for their parents: \(\partial (\Pi_1 + \Pi_2) / \partial \alpha > 0\) for all \(\alpha \leq 8/9\); that is, a larger \(\alpha\) raises the joint profits as long as firms do not engage in drastic price deviations. It does this primarily by effecting the JV’s pricing decisions. Clearly, different levels of \(\alpha\) do not really effect the incentives of parents to cut prices to steal sales from
the JV. However, the more cooperative the JV is (i.e. the larger is \( \alpha \)) the more “buffered” its competition with its parents is from its side: the extra profits the JV might earn through lowering prices and stealing market share from its parents need to be balanced against the harm done to its parents’ profits which matter to the JV more when \( \alpha \) is larger.

Comparing again with the pre-JV case, we see that the JV does not necessarily raise its parents’ profits. At \( \alpha = 1/2, \Pi_1 + \Pi_2 = \Pi_1^G + \Pi_2^G \), where \( \Pi_1^G + \Pi_2^G = tM \) is the level of joint profits in the two-good benchmark. Hence, once we consider the fixed costs of setting up the JV, we see immediately that this JV will never be profitable for its parents when \( \alpha < 1/2 \) and can only be profitable when \( \alpha > 1/2 \), if the higher levels of profits before fixed costs \( (\Pi_i - \Pi_i^G) \) are large enough to offset the (shared) fixed costs, i.e. if \( \Pi_i - \Pi_i^G > F/2 \).

We are now in a position to determine the conditions under which the firms will choose to establish a JV.

**Proposition 1:** Assume all firms choose prices simultaneously. For a given level \( \alpha \in \left[0, \frac{8}{9}\right] \), Firm 1 and Firm 2 will find it profitable to set up a joint venture if

\[
F < \frac{(2\alpha - 1)(36 - 23\alpha)Mt}{4(5 - 3\alpha)^2}.
\]

This condition requires a low enough level of fixed costs and a high enough level of cooperative behavior (\( \alpha \)) by the JV for the JV to be profitable for the parents. For example, as noted above, even if fixed costs were zero, the value of \( \alpha \) would have to be great than \( 1/2 \) for this to be satisfied.

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18 It is straightforward to show that a third firm, if not prevented by factors outside our model (e.g. intellectual property protections) would find it profitable to enter this market if \( F < Mt/4 \). It can also be shown that if fixed costs are this low, it would be more profitable for the parents to create a JV as a blocking strategy – even a fully independent JV will yield greater profits for parents than they would receive if there was new entry. These are demonstrated in the Appendix where, as a further benchmark, the implications for prices, profits and welfare of entry by an independent third firm are developed. Importantly, behaviors of the sort described by Schmalensee (1978) would be present here if new entry were permitted.

19 Proofs of all propositions are contained in the Appendix.
The maximum value $\alpha$ can take without leading to mixed strategies (i.e. $\alpha = 8/9$) would allow for profitable JVs as long as $F \leq (5/11)Mt$. 20

Next, we consider the welfare consequences of JV under the assumption that $\alpha \in [0, 8/9]$. To express the welfare measures in the situation where three goods are sold in equilibrium, let $\hat{\theta}$ denote the location of the consumer who is indifferent between good A and good C, and $\bar{\theta}$ the location of the consumer indifferent between good B and good C. The aggregate consumer surplus can be expressed as

$$CS' = M \left[ V - p_A\hat{\theta} - p_B(1 - \bar{\theta}) - p_C(\bar{\theta} - \hat{\theta}) - \frac{t}{2}\hat{\theta}^2 - \frac{t}{2}(1 - \bar{\theta})^2 - \frac{t}{2}(\frac{1}{2} - \hat{\theta})^2 - \frac{t}{2}(\frac{1}{2} - \bar{\theta})^2 \right]. \quad (15)$$

The total surplus associated with this case is:

$$TS' = M \left[ V - \frac{t}{2}\hat{\theta}^2 - \frac{t}{2}(1 - \bar{\theta})^2 - \frac{t}{2}(\frac{1}{2} - \hat{\theta})^2 - \frac{t}{2}(\frac{1}{2} - \bar{\theta})^2 \right] - F. \quad (16)$$

We substitute the equilibrium values of $p_i$, $\hat{\theta}$, and $\bar{\theta}$ into (15) and (16) to obtain the equilibrium levels of consumer welfare and total welfare. First, for consumer surplus we have:

$$CS = M \left[ V - \frac{(154 - 84\alpha - 7\alpha^2)}{8(5 - 3\alpha)^2} t \right]. \quad (17)$$

It is easy to see that consumer surplus decreases monotonically as $\alpha$ increases: i.e. $\partial CS / \partial \alpha < 0$. This should not be surprising as we have already seen that prices rise with $\alpha$.

However, compared to the two-product benchmark there are two effects on consumer surplus that work in opposite directions. The higher prices under the JV is one effect, but there is also now a new product with the associated reduced transportation costs. Taking these together and working with (17) we can show that $CS > CS^O$ if $\alpha < (108 - 28\sqrt{3})/97(\approx 0.613)$, and $CS \leq CS^O$ otherwise. Recall that the JV is potentially profitable (for $F$ small enough) if $\alpha >$

---

20 Notice that this level of $F$ is much higher than the critical level at which a new entrant would chose to move into the space (see footnote 18), indicating that buffering JVs may arise under a wider set of parameters than simple entry-blocking JVs.
1/2, in part because it will increase prices above that level. Hence, for $\alpha \in (0.5, 0.613)$ the JV improves consumer welfare and is potentially profitable. In words, as long as $\alpha$ is not too large, the consumer benefits of having more products (and reduced transportation costs) are greater than the losses suffered from higher JV prices.

These results allow us to establish the following:

**Proposition 2**: Under simultaneous price-setting, and considering the range $\alpha \in [0, 8/9]$, a JV improves consumer welfare if it is sufficiently independent (i.e. approximately: $\alpha < 0.613$) and reduces consumer welfare otherwise.

In terms of total surplus, we find:

$$TS = M \left[ V - \frac{(26 - 34\alpha + 13\alpha^2)}{8(5 - 3\alpha)^2} t \right] - F. \quad (18)$$

Our assumption on a lower bound for $V$ ($V \geq (11/6)t$) guarantees that, in all cases we consider, the market will be covered (all potential buyers will buy some product). Combined with the 0-1 nature of demand this means that total surplus will be determined by the amount of transportation cost incurred by buyers and not by price levels. For a given number of products at set locations, total transportation costs will be minimized when all prices are the same – identical prices imply that everyone will buy from the closest product and this will minimize transportation costs. From (18) we can see that $\partial TS / \partial \alpha > 0$ if $\alpha < 1/2$, and $\partial TS / \partial \alpha < 0$ if $\alpha > 1/2$. These relationships are explained by the fact that prices are equal when $\alpha = \frac{1}{2}$ and become more different the further $\alpha$ gets from $\frac{1}{2}$.

Following this, $TS$ achieves its maximum at $\alpha = 1/2$. Its minimum is at $\alpha = 8/9$. For $\alpha$ in this range, the JV improves total surplus if $F$ is small enough. This leads to:
**Proposition 3:** Assume all firms choose prices simultaneously and consider the range $\alpha \in [0, 8/9]$. The impact of a JV on total surplus depends on the magnitude of $F$.

Specifically, the JV will raise total surplus if

$$F < \frac{(6 - 5\alpha)(4 - \alpha)Mt}{8(5 - 3\alpha)^2}$$

For example, in the case of an independent JV ($\alpha = 0$) we would need $F < (3/25)Mt$ for the JV to raise total surplus. The critical $F$ would be a little higher when $\alpha = 1/2$ and prices are equalized, that is we would need $F < Mt/8$ for total surplus to increase. When $\alpha = 8/9$, the critical $F$ is at its lowest, $F < Mt/9$.

Again, Proposition 3 reflects the impact of two opposing forces on total surplus. First, the additional product introduced with the JV reduces average transportation costs for consumers. However, this gain in social welfare can be overwhelmed by the additional fixed cost if the latter is too large.

Consider now the implications of allowing the players to structure the JV governance as they see fit. Suppose that Firm 1 and Firm 2 actually select their parameter, $\alpha$, at stage 1 when they decide to set up the JV. Clearly, they will choose a value of $\alpha$ that maximizes their profits. From (14), we can see that the profit of each parent firm rises with $\alpha$ in the range $[0, 8/9]$. Hence, they would choose $\alpha = 8/9$ as long as the profit is higher at this value of $\alpha$ than that in a mixed-strategy equilibrium that would prevail if $\alpha > 8/9$. The latter, as shown in the Appendix, is indeed true.\(^2\)

**Proposition 4:** Assume all firms will choose prices simultaneously. If Firm 1 and Firm 2 can chose the level of $\alpha$ to be used in the JV’s objective function, they will set up a JV with $\alpha = 8/9$ as long as $F < (5/9)Mt$. With such a JV, consumer surplus is reduced relative to

\(^2\)It is perhaps not surprising that a mixed-strategy equilibrium would generate a smaller profit. When $\alpha > 8/9$, the JV has an incentive to raise its price to such a high level that makes itself vulnerable to drastic price cuts by the parent firms. In a mixed-strategy equilibrium, a parent firm indeed undercuts and steals the whole market share from the JV with some probability. This makes a mixed-strategy equilibrium less profitable than the most profitable pure-strategy equilibrium (which occurs at $\alpha = 8/9$).
the two-product benchmark. Total surplus relative to the two-product benchmark, will also decrease unless fixed costs are such that \( F \leq Mt/9 \).

In this section, we have seen the potential for JVs to reduce competition between parents to the point of harming consumers and reducing total surplus, even though they introduce a new product into the market. Partners who argue to competition authorities that their JV must be good for consumers because it involves the introduction of a new product are telling only part of the story of the JV’s effect on the market. Of course, we have also seen that JVs can, under some circumstances – particularly when operated independently from their parents – improve consumer welfare and raise total surplus.

The next section asks the same set of questions about the effects of the introduction of a new JV, but for the case in which the parents agree to let the JV be a price leader in the market.

V. Joint Venture as Price Leader

We now turn to the case in which the JV is a price leader in the market. It would seem reasonable to believe that the parents, in coming together to create the JV might also come to an understanding that they will follow the pricing lead of their JV. As we show, it is in fact profitable for them to adopt this behavior.

In this scenario, the JV chooses the price of good C before the parent firms set the prices of goods A and B. Accordingly, the JV maximizes its objective function given in (10) subject to the parent firms’ reaction functions, (8) and (9). Solving this optimization problem, we find:

\[
 p_C = \frac{2(3 + \alpha)t}{(4 - 3\alpha)}. \tag{19}
\]

Substituting the above into (8) and (9) gives us

\[
 p_A = p_B = \frac{(22 + 3\alpha)t}{4(4 - 3\alpha)}. \tag{20}
\]

Recall that (8) and (9) are applicable only for \( p_C \leq \left( \frac{5}{3} \right) t \). Imposing this restriction on (19), we find that \( \alpha \leq 2/21 \approx 0.095 \). In other words, (19) and (20) give the equilibrium prices for \( \alpha \in [0, 2/21] \).
If \( \alpha > 2/21 \), on the other hand, (19) would imply \( p_C > \left( \frac{5}{3} \right) t \). Given such a high \( p_C \), each parent will engage in a drastic deviation in an attempt to capture the JV’s entire market share. But as both parents do this, they end up competing directly against each other, leading to the same equilibrium prices and quantities as in the two-good benchmark. Anticipating this, the JV -- now a price leader -- will choose \( p_C = \left( \frac{5}{3} \right) t \), the highest price without inducing drastic price deviations. This leads the parents to choose \( p_A = p_B = \left( \frac{3}{2} \right) t \) as per (8)-(9). These then are the equilibrium prices in the case \( \alpha \geq 2/21 \). Notice, that putting the JV into the price leadership position avoids the need to consider mixed strategies as we had to with simultaneous price-setting and values of \( \alpha \) approaching one.

Comparing the prices in (19) and (20) with those in the two-product benchmark, we can see that all prices are higher with the JV: \( p_i > p_i^0 \) for all \( \alpha \in [0, 2/21] \). Moreover, the more cooperative the JV, the higher all three prices will be up to point at which higher JV prices would trigger drastic price cuts from parents, i.e. \( \partial p_i / \partial \alpha > 0 \forall i \), in the range \( \alpha \in [0, 2/21] \). For values of \( \alpha > 2/21 \), all prices will stay at the constrained levels (\( p_C = \left( \frac{5}{3} \right) t, p_A = p_B = \left( \frac{3}{2} \right) t \)). Note that these prices of goods A and B are higher than those in the two-product benchmark.

Returning to the case \( \alpha \leq 2/21 \), we substitute (19) and (20) into (1) to find the equilibrium quantities:

\[
Q_A = Q_B = \frac{(10 - \alpha)M}{8(4 - 3\alpha)}, \quad Q_C = \frac{(6 - 11\alpha)M}{4(4 - 3\alpha)}. \quad (21)
\]

Over the range in which increasing \( \alpha \) changes prices, more cooperate JVs will result in larger market shares for the parents and a smaller market share for the JV: \( \partial Q_A / \partial \alpha = \partial Q_B / \partial \alpha > 0 \) and \( \partial Q_C / \partial \alpha < 0 \) for all \( \alpha \in [0, 2/21] \). For independent JVs (\( \alpha = 0 \)), the JV will itself have 3/8 of the market sales while each parent will have 5/16 shares. When JVs are more cooperative, to the point that \( \alpha = 2/21 \) or above, each product (the JV’s included) will have a 1/3 market share.

Using (19)-(21), we obtain a parent firm’s profit:

\[
\Pi_i - \frac{F}{2} = \frac{(364 - 208\alpha - 91\alpha^2)Mt}{32(4 - 3\alpha)^2} - \frac{F}{2}, \quad (i = 1, 2). \quad (22)
\]
Profits of both parents rise if the JV is more cooperative, up to the point where prices stop changing (i.e. once \( p_c = \left( \frac{5}{3} \right) t \)): i.e. \( \partial \Pi_i / \partial \alpha > 0 \) for \( \alpha \leq 2/21 \).

For all values of \( \alpha \), gross profits before considering fixed costs are higher with the JV than in the two-product benchmark, i.e. \( \Pi_i > \Pi_i^0 \) for all \( \alpha \in [0,1] \). Hence, setting up this type of JV is always profitable for a small enough \( F \), i.e., if \( F/2 < \Pi_i - \Pi_i^0 \). This leads to the following:

**Proposition 5**: Assume the joint venture sets its price before the parent firms select theirs. For a given level of \( \alpha \in [0, 2/21] \), Firm 1 and Firm 2 will find it profitable to set up a joint venture if

\[
F < \frac{(108 + 176\alpha - 235\alpha^2)Mt}{16(4 - 3\alpha)^2}.
\]

For \( \alpha \in [2/21, 1] \), the JV raises total surplus if \( F < \left( \frac{5}{9} \right) Mt \).

Notice that this is a weaker condition than that for the simultaneous price-setting case. To see this notice that, whereas no JV with \( \alpha < \frac{1}{2} \) could be profitable in the simultaneous case, now even if the JV is fully independent (\( \alpha = 0 \)), it can be profitable if \( F < (27/64)Mt \).

Compared with the JV under simultaneous price-setting, the price-leadership JV achieves the same level of equilibrium profit with a much smaller \( \alpha \) (2/21 rather than 8/9). Firms may prefer this type of JV if a larger \( \alpha \) attracts the scrutiny of competition authorities. Reinforcing this point, note that the firms can achieve the same levels of prices and profits with an independent (\( \alpha = 0 \)) price-leadership JV as those under a simultaneous price-setting JV with \( \alpha = 9/11 \) (\( \approx 0.818 \)).

Again, we substitute the equilibrium values of \( p_l, \hat{\theta}, \) and \( \bar{\theta} \) into (15) and (16) to obtain the levels of consumer welfare and total surplus for \( \alpha \in [0, 2/21] \). For consumer surplus we have:

\[
CS = M \left[ V - \frac{(796 - 492\alpha - 121\alpha^2)}{32(4 - 3\alpha)^2} t \right]. \quad (23)
\]
The more cooperative the JV, the lower the level of consumer surplus: i.e. $\frac{\partial CS}{\partial \alpha} < 0$ up to $\alpha = 2/21$ (then CS does not change further for higher levels of $\alpha$). And, for any level of cooperativeness in the JV, consumer surplus with the JV is lower than consumer surplus with in the two-product benchmark; i.e. $CS < CS^O$ for all $\alpha \geq 0$. These observations give us:

**Proposition 6:** If the joint venture acts as a price leader, consumer welfare falls with the establishment of a JV for all values of $\alpha$.

Price leadership by the JV leads to higher prices for all products, sufficiently higher to outweigh any benefit to consumers from reduced distance to their preferred product. Prices are highest when the JV is fully cooperative.

For total surplus, the expression becomes:

$$TS = M \left[ V - \frac{(68 - 76\alpha + 61\alpha^2)}{32(4 - 3\alpha)^2} t \right] - F. \quad (24)$$

Here we can see that the more cooperative the JV, the lower the level of total surplus; i.e. $\frac{\partial TS}{\partial \alpha} < 0$. Total surplus is the highest at $\alpha = 0$ and the lowest at $\alpha = 2/21$ and above. Whether any JV can increase total surplus under price leadership will again depend on the level of fixed costs.

**Proposition 7:** Assume the joint venture acts as a price leader. A joint venture will increase total surplus if fixed costs are low enough, specifically if, for $\alpha \in [0, 2/21)$:

$$F < \frac{(60 - 116\alpha + 11\alpha^2)Mt}{32(4 - 3\alpha)^2}.$$  

For $\alpha \in [2/21, 1]$, the JV raises total surplus if $F < Mt/9$.

As we did in the previous case, we can consider the implications of allowing the players to structure the JV governance as they see fit. If we allow the JV partners to actually select their parameter, $\alpha$, it is clear from the above what they will do. At $\alpha = 2/21$, $\Pi_t = \left(\frac{7}{9}\right)Mt$; it is the
highest level of gross profit that a parent firm can achieve with this JV (before attracting drastic price deviations). Therefore, in the equilibrium of the whole game, $\alpha = 2/21$, $\Pi_1 = \Pi_2 = (7/9)Mt$, and setting up the JV is profitable if $F < (5/9)Mt$.

**Proposition 8**: Assume the JV acts as a price leader. If Firm 1 and Firm 2 can chose the level of $\alpha$ to be used in the JV’s objective function, they will set up a JV with $\alpha \geq 2/21$ as long as $F < (5/9)Mt$. With this choice, relative to the two-product benchmark, consumer surplus is reduced. Total surplus relative to the two-product benchmark, will also decrease unless fixed costs are such that $F \leq Mt/9$.

We can see the profitability of price leadership by comparing the profits in the simultaneous move case (14) with those from the price leadership case here (22).

**Proposition 9**: The combined profits of the parents under JV price leadership are weakly higher, for all values of $\alpha$, than combined profits with a JV under simultaneous price-setting and are strictly higher for all values of $\alpha < 8/9$.

Not surprisingly, price leadership, by pulling prices further up, increases profits for all values of the $\alpha$ parameter. Figure 1 illustrates the relationship between per-parent profits and the parameter $\alpha$, for the simultaneous move and price-leadership cases. We can see there that only at the level $\alpha = 8/9$ are profits the same in both cases.

Similarly, we can, compare the levels of consumer surplus and total surplus under simultaneous choices and price leadership. For consumer surplus this involves comparing levels from (17) and (23). For total surplus we need to compare the levels given by (18) and (24).

**Proposition 10**: For values of $\alpha$ in the range $\alpha \in [0, 8/9)$ consumer surplus and total surplus are strictly lower in the JV price leadership case than when all firms set prices simultaneously. When $\alpha = 8/9$, consumer surplus is the same in the two cases as is total surplus.
Figure 2 illustrates the values taken by these surplus functions for different values of $\alpha$ separately for the simultaneous and price leadership cases. Combining these results we see that both the sequencing of moves and the level of cooperativeness ($\alpha$) in the JV influence its effects. In brief terms, JV price leadership and more cooperative JVs are more profitable but also more harmful to consumers and total surplus.

VI. JVs and Collusion

In the analysis above, we assumed that while the parent firms cooperated in establishing the joint venture and in setting its price, their pricing behavior with respect their original products was strictly non-cooperative. In fact, our main interest was in how the JV could serve to buffer this competition and lead to higher profits.

It is natural, however, to be concerned that competitors that meet to form joint ventures might agree to cooperate in other dimensions. In this section we want to explore the profitability of collusion with and without a JV to see if JVs can raise the gains to collusion and to assess the consequences for consumers and total welfare of collusion with and without JVs.

There has long been a concern in antitrust circles that cooperation of firms in one arena (e.g. in setting up a JV) could spill over into reduced competition in markets in which they are expected to compete. Various mechanisms generating this spillover have been hypothesized, including improved information flows between colluding firms, the cover of cartel meetings provided by the need to meet to discuss legitimate cooperation, the use of a JV as a vehicle for the punishment of defectors, and simply the building of trust through working together.

In this section we allow the parents to cooperatively set their two prices in the two-product benchmark and then all three prices when they have established their joint venture. Collusion with its associated total profit maximization then replaces the different governance structures (i.e. $\alpha$) we considered above.

Collusion without a JV

\[22\] To clearly illustrate the differences between these curves, the scales on the two panels are different (so the heights of curves are not comparable across panels).

\[23\] The question of whether the presence of a JV will affect the stability of collusion is beyond the scope of our static model here, but is one we are exploring in further work.

\[24\] For example, see Cooper and Ross (2009) and the literature cited therein.
Consider first collusion by the two firms when there is no joint venture. It can easily be shown that the cartel will set the price such that the cartel extracts the entire surplus of the consumer in the middle of the line: $p_i^{CO} = V - t/2$. As noted, given the assumption $V \geq \frac{11}{6}t$, the firms will not raise prices any higher (which would lead to a portion of the market not covered).

The collusive profit of firm $i (= 1, 2)$ in this scenario is:

$$\Pi_i^{CO} = \frac{M}{4}(2V - t),$$

where superscript $C$ indicates collusion (or cartel) and $O$ indicates the original benchmark case of two products.

### Collusion with a Joint Venture

Consider now a situation in which the firms set up a JV to produce good C located in the middle of the line and then decide whether to collude. Colluding firms will choose the prices of all three goods to maximize their joint profit. Accordingly, the resulting collusive prices are independent of the governance form of the JV—in effect JV governance has been supplanted by cartel governance.

It is straightforward to show that, given $V \geq \frac{11}{6}t$, the three-product cartel also has no incentive to raise the prices to the point where a portion of the market is not covered. Instead, it will set the prices of the three goods such that the marginal consumer between any pair of goods pays a delivered price equal to the consumer’s willingness to pay, $V$. This will involve identical prices for the three products. The collusive prices and quantities are:

$$p_i^{CJ} = V - \frac{1}{4}t; \quad Q_A^{CJ} = Q_B^{CJ} = \frac{1}{4}M, \quad Q_C^{CJ} = \frac{1}{2}M.$$  \hspace{1cm} (26)

Before deducting the fixed cost of establishing the JV, the collusive profit of each parent is:

$$\Pi_1^{CJ} = \Pi_2^{CJ} = \frac{1}{2}M \left( V - \frac{1}{4}t \right).$$  \hspace{1cm} (27)

### Comparison of Collusion with and without a JV

---

$^{25}$ While, as above, we will simply assume that the JV is located in the middle of the line, it can be shown that it will be profit maximizing for the colluding firms to locate it there.
Here we investigate the effects of JV on the profitability, consumer surplus and total welfare effects of collusion by comparing the cartel equilibrium in the presence of JV with that in the absence of JV. First, we examine the profitability of collusion. A comparison of $\Pi^C_J - 0.5F$ with $\Pi^C_O$ reveals

**Proposition 11**: Collusion with a JV is more profitable than collusion without a JV if, and only if, $F < Mt/4$.

This conditional result should not be surprising. Since the cartel price is pushed up to the level at which the customer most distant is indifferent about consuming, it is effectively constrained by distance and transportation costs. Putting another product in the middle of the space reduces those costs, allowing room for the cartel to raise prices further. However this comes at the cost of introducing the new product, F. If F is too large, in this case if $F > Mt/4$, the higher prices do not justify introducing the JV.26

Next, we compare the welfare levels of collusion with and without a JV. If the two firms collude without using a JV, the level of consumer welfare is:

$$CS^{CO} = \frac{1}{4} Mt.$$ (28)

The level of total surplus is the same as in (6), as the market remains covered and transportation costs are unchanged.

If the firms collude in the presence of a buffer JV, the levels of consumer and total surplus are:

$$CS^{CJ} = \frac{1}{8} Mt.$$ (29)

$$TS^{CJ} = M \left[ V - \frac{1}{8} t \right] - F.$$ (30)

Comparing (29)-(30) with (28) and (6), leads to the following:

**Proposition 12**: For consumers, collusion with a JV is worse than collusion without JV.

---

26 Recall (from footnote 18) that $F < Mt/4$ is also the condition that would make entry by an independent third firm profitable (absent collusion).
While consumers benefit from reduced transportation costs with the JV, they are hurt by the higher prices they face. On balance here they are made worse off by the introduction of the JV.

*Proposition 13:* Collusion with a JV is better for total surplus than collusion without a JV if and only if \( F < \frac{Mt}{8} \).\(^{27}\)

Summarizing, we have seen that buffer JVs can have significant implications for the effects of collusion in our model. Specifically we have found that:

(i) for low levels of the fixed cost, collusion with a JV will be more profitable than collusion without the JV;

(ii) collusion with a JV is always worse for consumers than collusion without a JV; and

(iii) relative welfare under collusion with a JV compared to collusion without a JV depends on the magnitude of fixed costs: for low values of \( F \), collusion with the JV is better for total surplus than collusion with no JV, but the reverse is true for higher levels of fixed costs.

**Joint Ventures vs. Collusion as Alternative Strategies**

We have seen in our non-cooperative game that JVs can reduce competition and raise parents’ profits. Of course, collusion can do this too. A final question of some interest then, is whether a JV is a good substitute strategy for parents contemplating collusion. Consider two parents choosing between collusion (without a JV) and a price-leading cooperative JV with \( \alpha \in [2/21, 1] \) (the most profitable case in our static setting). While it might not achieve the same level of prices and it will involve incurring an additional fixed cost, the JV may avoid legal risks associated with cartel conduct.

A full analysis of this case would require a model with antitrust enforcement, which is beyond the scope of this paper. However, we can at least see how close the JV profits will be to the full collusive profits. Recall that the most profitable JV equilibrium occurs at prices \( p_A = \)

\(^{27}\) It can easily be shown that \( F < \frac{Mt}{8} \) is also the condition for entry by an independent third firm to be total surplus increasing (absent collusion). An implication of this proposition is that, when fixed costs are great enough that new entry is not a concern (i.e., \( F > \frac{Mt}{4} \)), a JV aggravates the total surplus loss from collusion.
\[ p_B = \left(\frac{\frac{3}{2}}{2} \right) t \text{ and } p_C = \left(\frac{\frac{5}{3}}{3} \right) t. \]
Comparing the gross profit (before deducting the fixed cost of the JV) at these prices with the two-good collusive profit given in (25), we obtain the following result.

**Proposition 14.** A price-leading cooperative JV with \( \alpha \in [2/21, 1] \) generates a larger gross profit for the two firms than collusion with two products if \( V \in \left(\frac{\frac{11}{6}}{6} t, \frac{\frac{37}{18}}{18} t\right) \). The opposite is true if \( V > \left(\frac{\frac{37}{18}}{18} t\right) \).

Collusion does allow for higher prices, but for lower values of \( V \) the possible increases are not as large. Introducing the JV does require new expenditures on fixed costs, but provides more freedom to raise prices by reducing transportation costs. Interestingly, the JV can be a more profitable route to higher profits than two-product collusion. On top of this, the JV might bring with it a lower antitrust risk for the parents.

**VII. Policy Implications and Conclusions**

In this paper we have described a new mechanism through which certain kinds of strategic alliances or joint ventures can threaten competition. The key idea is that a new entity, jointly owned, placed between two parents in product (or geographic) space can serve to buffer the competition between the parents. Now each parent, for at least some of its customers, is competing with the JV -- which it owns jointly with the other parent. Recognizing their own ownership positions in the JV will soften parents’ incentives to compete, which will be manifest in their own pricing decisions as well as in their design for the JV. We have demonstrated the potential for anticompetitive outcomes in an otherwise competitive environment and we have seen that the presence of JVs can affect the gains to collusion.

We have shown how the anticompetitive effects depend on the governance structure of the joint venture. Independent joint ventures, in which the JV chooses its price to maximize its own profit, tend not to be as profitable for the parents as more cooperative joint ventures which set prices to maximize total profits.

We have also shown that the effects of the JV will depend on whether the parents let it be a price leader as opposed to choosing price at the same time as they do. Price leadership by the
JV tends to be more profitable for the parents and more harmful to consumers as it results in even higher prices. Consumer surplus under an independent JV will increase if prices are set simultaneously, however consumer surplus will fall under an independent JV that is a price leader and in both simultaneous and leadership model with more cooperative JVs.

The introduction of a joint venture can have implications for the effects of collusion as well. Collusion with a JV can be more profitable than collusion without a JV if fixed costs are low enough, and collusion with a JV is always worse for consumers (than collusion without a JV). This suggests a kind of complementarity between JVs and collusion. Importantly, however, they can also be substitutes: we show that adding a cooperative JV can, in some cases, be as profitable a strategy for parents as them jointly setting their two prices – and with lower risk of running afoul of antitrust laws.

Most of the effects described here will depend to some extent on the fixed costs associated with introducing a new product. This is intuitive, of course: if fixed costs are high enough, no type of JV can be profitable. As we consider lower levels of fixed costs, however, we observe a number of critical values that determine what arrangements increase or decrease profits, consumer surplus and total surplus.

Importantly, these results are not driven by forces described in the earlier literature. For example, they do no derive from the JV “filling gaps” and thereby preventing entry by a new player – though this effect can arise here as well, adding to the effects we study. We show that, even if there is no threat of new, independent entry, there are conditions under which parents will still want to create a buffer JV to influence the competition between themselves. The results are also not due to granting the parents direct control over each other (as with cross-ownership). In setting their own produce prices in the static game, each parent simply maximizes its own profit – though this does now include some share of the profits of the new JV.

The model developed here suggests some factors competition authorities might want to consider when evaluating a joint venture for its anticompetitive potential. First, where does the JV lie in relation to its parents in product or geographic space – is it in a buffering position? Second, how is the JV governed – does it run as an independent operation, at arms-length from its parents, or do the parents jointly control key strategic decisions, for example over price? Third, does it appear that the JV has been positioned as the price leader in the market? Fourth, an examination of pricing levels after the creation of the JV might suggest that the JV has
enabled the parents to reach less competitive outcomes, possibly even leading them to explicitly collude.

The model here is highly stylized and a number of directions for future research suggest themselves. First, our Hotelling set-up combined with assumptions assuring market coverage has meant that there were no output effects (with the associated deadweight loss) from higher prices. A more general model of differentiation would allow for quantity effects and would, we conjecture, provide results in which JVs were even more harmful to total welfare than they are here. Pushing the model of differentiation further, we could allow firms to compete over more than just price and location. In the spirit of Winter (1993), for example, we could add an additional aspect of differentiation which would open up the question of where the parents would choose to locate their JV. We could also, of course, consider other cost functions – for example quadratic transportation costs.

Finally, we could consider the implications of joint ventures for the stability of collusion between parents. Preliminary work has suggested that, while the presence of a JV between the parents does have a buffering effect reducing the gains from defection for any given collusive price, the higher prices made possible with the JV filling the product space make for fatter profit margins and stronger incentives to deviate. The net effect of these opposing forces will determine whether cartels with JVs are more or less stable.
References


Figure 1: Parent Gross Profits

Figure 2

Panel (a): Consumer Surplus

Panel (b): Total Surplus
Appendix

A1. Entry by an Independent Third Firm

Here we investigate the conditions under which a third firm might want to place a product between A and B. To do so, we consider the following two-stage game. At stage 1, the potential entrant decides whether to enter the market by offering good C (located at $x = 0.5$). At stage 2, the three firms choose their prices simultaneously.

Recall that the demand functions for the three goods are represented in (1). Using these demand functions, we solve the profit-maximization problems of the three firms and find their best-response functions:

$$p_A = \frac{t + 2p_C}{4}. \quad (A1)$$

$$p_B = \frac{t + 2p_C}{4}. \quad (A2)$$

$$p_C = \frac{t + p_A + p_B}{4}. \quad (A3)$$

Solving the equation system (A1)-(A3), we find the equilibrium prices at stage 2:

$$p_A^E = p_B^E = p_C^E = \frac{t}{2}. \quad (A4)$$

Substituting (A4) into the demand functions in (1), we obtain the entrant’s equilibrium quantity $Q_C^E = M/2$. Using the equilibrium price and quantity, we find the entrant’s gross profit $\pi_C^E = Mt/4$. Hence, the entrant would choose to enter the market if $\pi_C^E \geq F$, i.e., if $F \leq Mt/4$. In this scenario, the profit of each incumbent is $\Pi_1^E = \Pi_2^E = Mt/8$. Using (15) and (16), we calculate the consumer surplus and total surplus in this scenario: $CS^E = M(V - 5t/8)$ and $TS^E = M(V - t/8) - F$. Comparing the total surplus in this scenario with that in the two-product benchmark, we see that the entry of a third product -- by lowering average travel costs -- will increase total surplus as long as $F < Mt/8$. Therefore, in the range $Mt/8 < F < Mt/4$ independent entry could happen that would be inefficient.

A2. Condition that Leads to Drastic Price Deviations

In this section, we demonstrate that a parent firm will engage in drastic price deviations if the JV sets its price above $5t/3$. A drastic price deviation occurs when a parent firm makes such a large price cut that it will steal the entire market share of the JV and face direct competition from the other parent firm. Specifically, a parent firm can achieve this by cutting its price below $p_C - t/2$, where $p_C$ is the price of JV’s product. In this scenario, the demand for the product of the deviating parent is given by (2) instead of (1).

Without loss of generality, we consider a drastic price deviation by firm 1. Given that this is a unilateral price deviation, firm 2 continues to set its price in accordance with its best-response function (9). Therefore, the (drastic) deviation price chosen by firm 1 is the solution to
\[
\max_{p_A} \Pi_1 = \pi_A + \frac{1}{2} \pi_C = p_A \left[ 1 - \frac{p_A - p_B}{2t} \right] \quad \text{s.t. } p_B = \frac{t + 3p_C}{4} \text{ and } p_A \leq p_C - \frac{t}{2}. \quad (A5)
\]

Note that \( \pi_C = 0 \) in (A5) because firm 1 steals the entire market share from the JV.

Let \( \Pi_1^D \) denote the (constrained) maximum of (A5). It turns out that the solution to (A5) depends on \( p_C \). If \( p_C \geq (9/5)t \), the inequality constraint in (A5) is slack, in which case firm 1’s deviation payoff is

\[
\Pi_1^D = \frac{M(3p_C + 5t)^2}{128t}. \quad (A6)
\]

On the other hand, firm 1’s gross profit (for any given \( p_C \)) from following its best-response function (8) is

\[
\Pi_1^{BR} = \frac{M(t^2 + 16tp_C - p_C^2)}{32t}. \quad (A7)
\]

A comparison of (A6) with (A7) shows that \( \Pi_1^D > \Pi_1^{BR} \) for \( p_C \geq (9/5)t \); in other words, firm 1 will have incentives to engage in drastic price deviation if the price set by the JV exceeds \((9/5)t\).

If \( p_C < (9/5)t \), the inequality constraint in (A5) is binding, in which case firm 1’s deviation payoff is

\[
\Pi_1^D = \frac{M(p_C - t/2)(7t - p_C)}{8t}. \quad (A8)
\]

Comparing (A8) with (A7), we find that \( \Pi_1^D > \Pi_1^{BR} \) for \( p_C \in ((5/3)t, (9/5)t) \) and \( \Pi_1^D \leq \Pi_1^{BR} \) for \( p_C \leq (5/3)t \). Therefore, firm 1 (and by the same reasoning, firm 2) will engage in drastic price deviation if and only if \( p_C > (5/3)t \).

A3. Mixed Strategy Equilibrium under Simultaneous Price-Setting

In section IV, we have explained that if \( \alpha > 8/9 \), only a mixed-strategy equilibrium is possible under simultaneous price-setting. Because this game has more than two players and has nonlinear payoff functions, we are not able to find a closed form solution for the equilibrium mixed strategies. Instead, we will derive an upper bound for the expected profit of a parent firm in a mixed-strategy equilibrium.

Let \( F_i(p_i) \) denote the cumulative distribution function of a firm’s prices, with subscript \( i = A \) (respectively, \( B \) and \( C \)) indicating firm 1’s (respectively, firm 2’s and JV’s) prices. In a mixed-strategy equilibrium, a player earns the same expected payoff from playing any pair of pure strategies within its price distribution, and it would earn a lower expected payoff from playing a pure strategy outside its price distribution. We use these two properties of a mixed-strategy equilibrium to delineate the support of \( F_i(p_i) \).

Let \([p^l_i, p^h_i]\) be the support of product \( i \)’s price \((i = A, B, C)\) in a mixed strategy equilibrium. Given that \( \alpha > 8/9 \), the JV’s best-response function in (12) prescribes a price \( (p_C) \) that exceeds \((5/3)t\), the threshold above which each parent firm has incentives to engage in drastic price deviations. At \( p_C = (5/3)t \), the best-response of a parent firm is to set \( p_i = (3/2)t \) \((i = A, B)\) as per (12). This implies that \( p^h_i \leq (3/2)t \) \((i = A, B)\) because starting from this point a parent firm will find it profitable to cut its price drastically in response to an increase in \( p_C \).
From (12), we also find that when \( p_A = p_B = (3/2)t \), the best-response price of the JV would have been \( p_C = [(4 + 3\alpha)/4]t \) in the absence of drastic price deviations. This implies that \( p_C^h \leq [(4 + 3\alpha)/4]t \) because any higher price will only reduce the JV’s expected payoff. Note that \((4 + 3\alpha)/4 \leq (7/4)t \) for \( \alpha \leq 1 \). Hence, \( p_C^h \leq (7/4)t \) for any \( \alpha \in (8/9, 1] \).

Note that at \( p_C = (5/3)t \), a parent firm needs to lower its price to \( p_i = (5/3)t - t/2 = (7/6)t \) in order to drastically undercut the JV’s price. Below we show that \( p_i^L \geq (7/6)t \) for \( i = A, B \) and \( p_C^L \geq (5/3)t \).

Without loss of generality, we use firm 1 as a representative of the parent firms. Suppose \( p_i^L < (7/6)t \), and consider the parent firm’s expected payoff from choosing a price \( p_A \in [p_i^L, (7/6)t] \). At such a low price, firm 1 competes directly with the other parent firm. Hence, its expected profit is
\[
E(\Pi_1) = \frac{F}{2} - \int_{p_B^l}^{p_B^h} M_A \left( \frac{1}{2} + \frac{p_B - p_A}{2t} \right) dF_B(p_B) - \frac{F}{2}.
\]

For firm 1 to randomize its price over \( [p_i^L, (7/6)t] \), the value of (A9) has to be constant for \( p_A \) in this range. This entails
\[
\frac{\partial E(\Pi_1)}{\partial p_A} = \int_{p_B^l}^{p_B^h} M \left( \frac{1}{2} + \frac{p_B - 2p_A}{2t} \right) dF_B(p_B) = 0 \quad (A10)
\]
for any \( p_A \in [p_i^L, (7/6)t] \). However, (A10) can be satisfied by only one value of \( p_A \), given by
\[
p_A = \frac{t + \int_{p_B^l}^{p_B^h} p_B dF_B(p_B)}{2}.
\]

In other words, firm 1 will not randomize its price over \( [p_i^L, (7/6)t] \), which contradicts the supposition that \( p_i^L < (7/6)t \). Hence, \( p_i^L \geq (7/6)t \).

We can use the same method to prove that \( p_C^L \geq (5/3)t \). Suppose \( p_i^L < (5/3)t \). Consider the JV’s expected payoff from choosing a price \( p_C \in [p_i^L, (5/3)t] \), which is
\[
E(\Pi_{JV}) = \int_{p_A^l}^{p_A^h} \int_{p_B^l}^{p_B^h} \Pi_{JV} dF_B(p_B) dF_A(p_A),
\]
where \( \Pi_{JV} \) is given in (10). For the JV to randomize its price over \( [p_C^L, (5/3)t] \), the value of (A12) has to be constant for \( p_C \) in this interval. This entails
\[
\frac{\partial E(\Pi_{JV})}{\partial p_C} = \frac{M}{2t} \int_{p_A^l}^{p_A^h} \int_{p_B^l}^{p_B^h} [t + (1 + \alpha)p_A + (1 + \alpha)p_B - 4p_C] dF_B(p_B) dF_A(p_A) = 0. \quad (A13)
\]
But there is only one value of \( p_C \) that satisfies (A13). This implies that the JV will not randomize its price over \( [p_C^L, (5/3)t] \), which contradicts the supposition that \( p_C^L < (5/3)t \). Hence, \( p_C^L \geq (5/3)t \).

To summarize, the preceding analysis shows that \( [p_i^L, p_i^h] \subset [(7/6)t, (3/2)t] \) for \( i = A, B \) and \( [p_C^L, p_C^h] \subset [(5/3)t, (7/4)t] \). We can further narrow the support of the parents’ prices by showing that \( p_i^h \leq (5/4)t \) for \( i = A, B \). We do so by using the same method as above. Specifically, suppose that \( p_A^h > (5/4)t \) and consider firm 1’s expected profit for \( p_A \in
((5/4)t, \( p^h_A \)). Note that \((7/4)t - t/2 = (5/4)t\). With \( p_A > (5/4)t \), firm 1 will not steal the JV’s entire market share (but firm 2 may). Thus, firm 1’s expected profit for \( p_A \) in this range is
\[
\frac{E(\Pi) - F}{2} = \int_{p_C}^{p_A} \int_{p_B}^{p_C} M p_A \left( \frac{t}{2} + \frac{p_B - p_A}{2t} \right) dF_B(p_B) dF_C(p_C) \\
+ \int_{p_C}^{p_B} \int_{p_C}^{p_B} \left[ M p_A \left( \frac{1}{4} - \frac{p_A - p_C}{2t} \right) + M p_C \left( \frac{1}{2} - \frac{2p_C - p_A - p_B}{2t} \right) \right] dF_B(p_B) dF_C(p_C) \\
- \frac{F}{2}.
\]
(A14)

Again, we can show that \( \frac{\partial E(\Pi)}{\partial p_A} = 0 \) is satisfied by only one value of \( p_A \), which implies that firm 1 will not randomize over \( p_A \in ((5/4)t, p^h_A) \). Hence, \( p^h_A \leq (5/4)t \).

Combining all the findings about each product’s price, we conclude that
\([p^l_i, p^h_i] \subset [(7/6)t, (5/4)t] \) for \( i = A, B \) and \([p^l_C, p^h_C] \subset [(5/3)t, (7/4)t] \). Next, we use this information to determine an upper bound on the payoffs of the parent firms in a mixed-strategy equilibrium. For this purpose, note that \( p^l_i - p^h_i = (5/12)t \) (\( i = A, B \)). This implies that \( p_C - p_i \geq (5/12)t \) and hence \( 2p_C - p_A - p_B \geq (5/6)t \) in a mixed-strategy equilibrium.

Consider the combined gross profits in a pure-strategy equilibrium,
\[
\Pi_1 + \Pi_2 = p_A \left( \frac{1}{4} - \frac{p_A - p_C}{2t} \right) M + p_B \left( \frac{1}{4} - \frac{p_B - p_C}{2t} \right) M \\
+ p_C \left( \frac{1}{2} - \frac{2p_C - p_A - p_B}{2t} \right) M.
\]
(A15)

Using (A15), we can verify that \( \frac{\partial (\Pi_1 + \Pi_2)}{\partial p_i} > 0 \) if \( p_C - p_i > -t/4 \) (\( i = A, B \)) and
\( \frac{\partial (\Pi_1 + \Pi_2)}{\partial p_C} < 0 \) if \( 2p_C - p_A - p_B > t/2 \). This implies that for prices in the ranges of \( p_i \in [(7/6)t, (5/4)t] \) (\( i = A, B \)) and \( p_C \in [(5/3)t, (7/4)t] \), the maximum value of (A15) is achieved at \( p_A = p_B = (5/4)t \) and \( p_C = (5/3)t \), which is equal to \((185/144)Mt\). This, in turn, implies that the expected combined gross profits in a mixed-strategy equilibrium is below \((185/144)Mt\). Taking into account the fixed costs, we conclude that the expected payoff of each parent firm in a mixed-strategy equilibrium is less than \((185/288)Mt - F/2\).

**A4. Proof of Propositions**

**Proof of Proposition 1.** The profit of a parent firm with the JV is given in (14), and its profits without the JV is given in (3). This proposition is obtained by comparing these two profit levels. In addition, we also prove here that the market is covered in equilibrium given the assumption that \( V > (11/6)t \). To see this, note that the highest prices that can be achieved without causing drastic price deviations are associated with \( \alpha = 9/8 \). The corresponding prices are \( p_A = p_B = (3/2)t \) and \( p_C = (5/3)t \). At these prices, the consumer who is indifferent between purchasing product A and product C is located at \( \theta = t/3 \). This consumer pays a delivered price of \((3/2)t + t/3 = (11/6)t\), which is the highest among all consumers along the Hotelling line. This consumer indeed purchases a unit in equilibrium given the assumption that \( V > (11/6)t \). Since equilibrium prices are lower for smaller values of \( \alpha \), this assumption ensures that the market is covered in equilibrium. QED
Proof of Proposition 2. The result follows from the comparison of consumer surplus in (17) with (5). QED

Proof of Proposition 3. The result follows from the comparison of total surplus given in (18) with (6). QED

Proof of Proposition 4. From (14), we know that $\frac{\partial \Pi_I}{\partial \alpha} > 0$ for all $\alpha \leq 8/9$. Hence, the highest level of profit in a pure-strategy equilibrium is achieved at $\alpha = 8/9$, which is $(7/9)Mt - F/2$. If the parent firms set $\alpha > 8/9$, a mixed-strategy equilibrium would prevail and, as shown in section A3, the expected payoff of each parent firm would be less than $(185/288)Mt - F/2$. Since $(7/9) > (185/288)$, they will set $\alpha = 8/9$ if they choose to establish the JV. Setting $\alpha = 8/9$ in Proposition 1, we find that establishing the JV is profitable if $F < (5/9)Mt$. The rest of this proposition follows from Propositions 2 and 3.

Proof of Proposition 5. For $\alpha \leq 2/21$, the result is obtained by comparing the profit levels in (22) and (3). For $\alpha > 2/21$, it follows from the comparison of $(7/9)Mt - F/2$ with (3). QED

Proof of Proposition 6. For $\alpha \leq 2/21$, the result is obtained by comparing the levels of consumer surplus in (23) and (5). For $\alpha > 2/21$, the consumer surplus is constant at $CS = M[V - (61/32)t]$ (which is obtained by setting $\alpha = 2/21$ in (23)). This is lower than the level of consumer surplus in (5). QED

Proof of Proposition 7. For $\alpha \leq 2/21$, the result is obtained by comparing the levels of total surplus in (24) and (6). For $\alpha > 2/21$, the total surplus is constant at $TS = M[V - (5/36)t] - F$ (which is obtained by setting $\alpha = 2/21$ in (24)). The result follows from comparing it with the level of total surplus in (6). QED

Proof of Proposition 8. From (14), we know that $\frac{\partial \Pi_I}{\partial \alpha} > 0$ for all $\alpha \leq 2/21$. For $\alpha > 2/21$, the equilibrium profit of a parent firm is constant at $(7/9)Mt - F/2$, which is the same level as at $\alpha = 2/21$. Therefore, the parent firms will set $\alpha \geq 2/21$ if they choose to establish the JV. The rest of this proposition follows from Propositions 5-7. QED

Proof of Proposition 9. The combined profits of the two firms are equal to 2 times the level given in (14) (under simultaneous price-setting) and (22) (under price leadership). It is straightforward to show that for $\alpha \leq 2/21$, the profit level in (22) is greater than that in (14). For $\alpha \in (2/21, 8/9]$, the profit level of each parent under price leadership is constant at $(7/9)Mt - F/2$. The profit of each parent under simultaneous price-setting, on the other hand, monotonically increases with $\alpha$ in this interval, reaching its highest level of $(7/9)Mt - F/2$ at $\alpha = 8/9$. This implies that the combined profits of the parents under JV price leadership are higher than combined profits with a JV under simultaneous price-setting for $\alpha \in (2/21, 8/9)$. QED
Proof of Proposition 10. It is straightforward to show that for $\alpha \leq 2/21$, the level of consumer surplus in (23) is lower than that in (17). For $\alpha \in (2/21, 8/9]$, the level of consumer surplus under price leadership is constant at $M[(V - (61/36)t]$. Using (14), we find that the level of consumer surplus under simultaneous price-setting monotonically decreases with $\alpha$ in this interval, with its level equal to $M[(V - (61/36)t] at $\alpha = 8/9$. This implies that consumer surplus under price leadership is lower than that under simultaneous price-setting for $\alpha \in (2/21, 8/9)$.

As for total surplus, we set $\alpha = 0$ in (18) and (24) to find that total surplus under simultaneous price-setting is higher than that under price leadership at this value of $\alpha$. Moreover, we differentiate (18) to find that under simultaneous price-setting,

$$\frac{\partial TS}{\partial \alpha} = \frac{7Mt(1 - 2\alpha)}{4(5 - 3\alpha)^3} \begin{cases} > 0 & \text{if } \alpha \in [0, 1/2) \\ < 0 & \text{if } \alpha \in (1/2, 8/9]. \end{cases} \quad (A16)$$

On the other hand, from (24) we find that under price leadership,

$$\frac{\partial TS}{\partial \alpha} = -\frac{13Mt(2 + 5\alpha)}{8(4 - 3\alpha)^3} < 0 \text{ for any } \alpha \in [0, 2/21]. \quad (A17)$$

Eq. (A16) and (A17) imply that for $\alpha \in [0, 2/21]$, total surplus rises under simultaneous price-setting but falls under price leadership. Hence, total surplus is higher under simultaneous price-setting than under price leadership for $\alpha \in [0, 2/21]$. For $\alpha \in (2/21, 8/9]$, total surplus under price leadership remains constant at the level of $\alpha = 2/21$. Under simultaneous price-setting, (A16) implies that total surplus first rises and then falls, and at $\alpha = 8/9$ it falls to the same level as the total surplus under price leadership. This implies that total surplus is higher under simultaneous price-setting than under price leadership for $\alpha \in (2/21, 8/9)$. QED

Proof of Proposition 11. Using (25) and (27), we find that collusion with the JV is more profitable than collusion without the JV if and only if

$$\frac{1}{2} M \left( V - \frac{1}{4} t \right) - \frac{F}{2} > \frac{M}{4} (2V - t). \quad (A18)$$

This implies $F < Mt/4$. QED

Proof of Proposition 12. The levels of consumer surplus in these two cases are given in (28) and (29). The result follows directly from the comparison of these two levels. QED

Proof of Proposition 13. The level of total surplus under collusion with two products is equal to that in (6), and that under collusion with the JV is in (30). The result follows directly from the comparison of these two levels. QED

Proof of Proposition 14. The gross profit for each parent under price-leading JV, calculated by setting $\alpha = 2/21$ in (22), is $(7/9)Mt - F/2$. From (25), we see that each firm’s profit under collusion with two products is $M (2V - t)/4$. The result follows from the comparison of these two profit levels. QED