

3. B. K. Sinha and N. Gopi, "Study of Thermal Relaxation in a Repetively Pumped Nd:Glass Laser Rod," *IEEE J. Quantum Electron.* **QE-16**, 433 (1980).
4. J. D. Foster and L. M. Osterink, "Thermal Effects in a Nd:YAG Laser," *J. Appl. Phys.* **41**, 3656 (1970).
5. W. Koefhner "Thermal Lensing in Nd:YAG Laser Rod," *Appl. Opt.* **9**, 2548 (1970).
6. J. M. Soares, S. Kumpan, and J. Hoose, "High Power Nd:Glass Laser for Fusion Applications," *Appl. Opt.* **13**, 2081 (1974).
7. W. Koehner, *Solid State Laser Engineering* (Springer-Verlag, New York, 1976), pp. 353.
8. J. F. Nye, *Physical Properties of Crystals* (Clarendon, New York, 1957), pp. 250–51.

Refractive-index profiles of planar waveguides made by ion-exchange in glass

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Optical waveguides made by ion-exchange in glass have attracted considerable interest lately as passive components in integrated optical circuits.¹⁻⁴ To design such waveguides and study their properties, it is necessary to have an accurate knowledge of the refractive-index profile resulting from this fabrication process. In particular, it is desirable to have an analytical model of the profile obtained from planar ion-exchange, from which waveguide parameters can be quickly determined. Such knowledge is also useful in characterizing channel waveguides and other structures (lenses, splitters, filters, etc., . . .) by the effective index method.⁵ The purpose of this Letter is to point out (by solving numerically the ion-exchange diffusion equation) that the well-established second-order polynomial model of the profile obtained for silver ion-exchange is not sufficiently accurate for potassium exchange. In the latter case, a Gaussian function provides a better model. This improved representation by a Gaussian function does not seem to have been explicitly pointed out by other workers.

Physically, the planar ion-exchange is described by the following nonlinear diffusion equation⁶ (in the depth x , measured from the surface of the substrate):

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(\frac{D_1}{1 - \alpha c} \frac{\partial c}{\partial x} \right), \quad (1a)$$

$$c = c_1/c_0 \quad \alpha = 1 - (D_1/D_0), \quad (1b)$$

where c_1 is the concentration of in-diffusing ions in the glass and D_1 is their self-diffusion coefficient, D_0 is the self-diffusion coefficient of the out-diffusing ions, and c_0 is the total concentration of diffusing ions in the glass (incoming plus outgoing). It must be constant to preserve neutrality. Finally t is the diffusion time.

In the following we use the transformation

$$y = x/(2\sqrt{D_1 t}) \quad (2)$$

to bring the equation to the more practical form

$$-2y \frac{dc}{dy} = \frac{d}{dy} \left(\frac{1}{1 - \alpha c} \frac{dc}{dy} \right); \quad c(\infty) = 0. \quad (3)$$

Note that the refractive-index change Δn is assumed to be proportional to the concentration of new ions in the glass:

$$\Delta n(y)/\Delta n_s = c(y)/c(0). \quad (4)$$

The only free parameters of Eq. (3) are α and the surface concentration $c(0)$. As was noted elsewhere,² the shape of the profile $c(y)$ obtained from Eq. (3) depends strongly on the value of α , varying from an error function complement for $\alpha = 0$ to a rectangular one for $\alpha = 1$. Therefore, different values of α will lead to different models for the index profile. Let us examine this effect with the two main ion species used to date in ion-exchange.

For silver-sodium exchange ($\text{Ag}^+ - \text{Na}^+$), for example, an exhaustive experimental study⁷ showed that a second-order polynomial was the best function to use for the index change in a WKB-type dispersion relation to match a set of measured effective indices. They confirmed this result by showing that the exact numerical solution of Eq. (3) could be well approximated for small y by an analytical power series solution truncated to the second order. In the process, they found the best value of α to be 0.56. In a more recent work,² a direct comparison between the mode spectrum obtained numerically from the exact solution of Eq. (3) and measured values yielded $\alpha = 0.7$.

On the other hand, when we consider the case of potassium ion-exchange ($\text{K}^+ - \text{Na}^+$) in a soda-lime glass, conditions become markedly different. The maximum index change Δn_s is an order of magnitude smaller, the self-diffusion coefficient of potassium is also much smaller, and the maximum concentration of exchanged ions is only a fraction H (around 0.9) of the total concentration c_0 .⁸ This last difference has the effect of changing the boundary condition at $x = y = 0$ from $c(0) = 1$ to $c(0) = H$.

In fact for temperatures near 375°K, values of α ranging from 0.977 to 0.998 for $\text{K}^+ - \text{Na}^+$ exchange have been cited.⁸ For these values and the new boundary condition, the solution of Eq. (3) changes drastically. It should be noted here that the self-diffusion coefficients also depend on the type of glass used as a substrate. The numbers quoted here are normal values for an ordinary commercial soda-lime glass.

To determine the effect of the new conditions on the index profile, Eq. (3) was solved numerically with an implicit unconditionally stable three-level finite difference scheme taking $\alpha = 0.998$ and $H = 0.9$. Then the index profile was calculated by the same second-order power series solution that was cited previously.⁷ The two resulting curves are shown in Fig. 1 along with the best Gaussian fit [$c(y) = c(0) \exp(-y^2/a^2)$] obtained by a linear least-squares fitting of the numerical result (solid curve). We see that the Gaussian fit reproduces much better the index profile than the polynomial model for which the range of validity is very small, being only near the origin. For comparison, these calculations were repeated for $\alpha = 0.56$ and $H = 1$ (typical of $\text{Ag}^+ - \text{Na}^+$ exchange). The results are plotted in Fig. 2. We see there that while the Gaussian fit is quite poor over a large portion of the waveguiding layer [for which $c(y) > 0$], the second-order polynomial fit follows the numerical solution fairly closely up to $y = 0.7$.

It must be noted here that due to their smaller index change, waveguides made with potassium are more weakly guiding. It is then more important to have a good fit near the tail of the profile since a significant portion of the power carried by the modes propagates there. In Stewart *et al.*,⁷ much of the data used in fitting the profile was obtained for well-guided modes which have most of their power concentrated near the maximum of the index profile so that a better

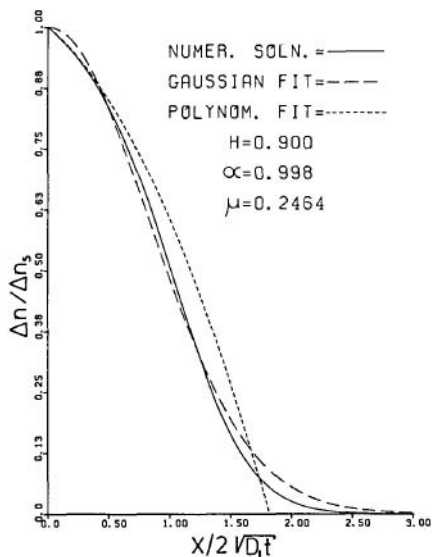


Fig. 1. Variation of refractive-index change with normalized depth for potassium ion-exchange. [μ is a parameter of the power series solution defined in Eq. (4.2.4) from Ref. 7.]

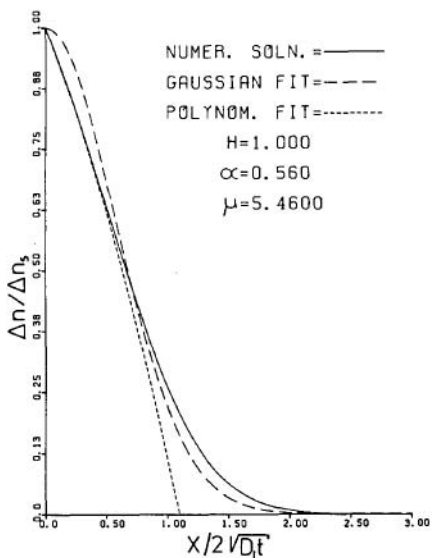


Fig. 2. Variation of refractive-index change with normalized depth for silver ion-exchange.

fit provided by the power series near the origin was an important factor.

To summarize: it has been shown that the shape of the refractive-index profile of ion-exchanged waveguides depends critically on the ion species and substrate materials involved through the parameter α , which depends in turn on the self-diffusion coefficients of the exchanging ions. In particular, through our rigorous numerical solution of the diffusion equation, we have demonstrated that for the K^+ - Na^+ exchange in soda-lime glass a Gaussian function provides a better model for the index profile than the second-order polynomial, which is more appropriate for Ag^+ - Na^+ exchange. Therefore, the use of the polynomial model should not be extended to cases other than Ag^+ - Na^+ exchange⁹ without careful examination of the characteristics of the specific process used. This fact is also confirmed by our recent experimental results on the characterization of potassium ion-exchanged waveguides.¹⁰

References

1. T. Findakly and B. Chen, "Single-Mode Integrated-Optical $1 \times N$ Star Coupler," *Appl. Phys. Lett.* **40**, 549 (1982).
2. R. G. Walker, C. D. W. Wilkinson, and J. A. H. Wilkinson, "Integrated Optical Waveguiding Structures made by Silver Ion-Exchange in Glass. 1: The Propagation Characteristics of Stripe Ion-Exchanged Waveguides; a Theoretical and Experimental Investigation," *Appl. Opt.* **22**, 1923 (1983).
3. R. G. Walker and C. D. W. Wilkinson, "Integrated Optical Waveguiding Structures made by Silver Ion-Exchange in Glass. 2: Directional Couplers and Bends," *Appl. Opt.* **22**, 1929 (1983).
4. K. Honda, E. M. Garmire, and K. E. Wilson, "Characteristics of an Integrated Optics Ring Resonator Fabricated in Glass," *IEEE/OSA J. Lightwave Technol.* **LT-2**, 714 (1984).
5. G. B. Hocker and W. K. Burns, "Mode Dispersion in Diffused Channel Waveguides by the Effective Index Method," *Appl. Opt.* **16**, 113 (1977).
6. F. Helfferich and M. S. Plesset, "Ion Exchange Kinetics. A Nonlinear Diffusion Problem," *J. Chem. Phys.* **28**, 418 (1958).
7. G. Stewart, C. A. Millar, P. J. R. Laybourn, C. D. W. Wilkinson, and R. M. DeLaRue, "Planar Optical Waveguides formed by Silver-Ion Migration in Glass," *IEEE J. Quantum Electron.* **QE-13**, 192 (1977).
8. R. H. Doremus, "Ion Exchange in Glasses," in *Ion-Exchange*, Vol. 2, J. A. Marinsky, Ed. (Dekker, New York, 1969).
9. Y. Okamura, S. Sato, and S. Yamamoto, "Simple Method of Measuring Propagation Properties of Integrated Optical Waveguides: an Improvement," *Appl. Opt.* **24**, 57 (1985).
10. G. L. Yip and J. Albert, "Characterization of Planar Optical Waveguides by K^+ -ion Exchange in Glass," *Opt. Lett.* **10**, 151 (1985).