

# STRESS-INDUCED INDEX CHANGE FOR $K^+-Na^+$ ION EXCHANGE IN GLASS

Indexing terms: Integrated optics, Optical waveguides

The index change and birefringence resulting from  $K^+-Na^+$  ion exchange in glass are shown to be mainly due to induced surface stresses. The importance of this fact with regard to the fabrication of optical waveguides is discussed.

Optical waveguides made by ion exchange in glass have been the focus of some interest in the past few years for making passive integrated-optical devices.<sup>1</sup> The purpose of this letter is to present a model of the index change that includes, for the first time, both the contribution of the different properties of the exchanging ions and the effect of the very large surface stresses induced by the process.<sup>2</sup> The latter effect proves to be dominant in the very important case of  $K^+-Na^+$  exchange.

The usual explanation for the index change resulting from ion exchange is based on the fact that the ions participating in the exchange have different electronic polarisabilities and that they occupy a different volume in the glass.<sup>1</sup> Quantitatively, a very accurate model has been developed<sup>3</sup> to predict the value of the index change which results from replacing one ion by another in the bulk composition of the glass:

$$\Delta n \approx \frac{\chi}{V_0} \left( \Delta R - \frac{R_0}{V_0} \Delta V \right) \quad (1)$$

where  $\chi$  is the fraction of the participating substrate ions that was actually exchanged (usually near 100%),  $V_0$  is the volume of glass per mole of oxygen atoms,  $R_0$  the refraction per mole of oxygen atoms (defined below) and  $\Delta V$  and  $\Delta R$  the changes in these quantities resulting from the ionic replacement. Using the tabulated values of Reference 3 and its eqn. 25 to obtain  $\Delta V$  and  $\Delta R$ , we obtain, for potassium-sodium ( $K^+-Na^+$ ) exchange in soda-lime glass,

$$\Delta n \approx 3 \times 10^{-4} \chi$$

This result is two orders of magnitude smaller than measured values (near 1%).<sup>1,4</sup> This conclusion also holds for the same ions in BK7 glass.<sup>5</sup> By contrast, a similar analysis for the case of silver-sodium ( $Ag^+-Na^+$ ) exchange in glass yields a  $\Delta n$  value of  $0.08\chi$ , in good agreement with the measured value<sup>6</sup> of 0.09. In the following, we describe how the discrepancy can be resolved when the large stresses<sup>2,7,8</sup> induced by ion exchange are included in the analysis.

These stresses come from the fact that at the low temperatures at which the process takes place (350–400°C for potassium), the glass is well below its strain point<sup>9</sup> (510°C), and the surface is prevented from expanding laterally by the resistance to bending of the relatively thick substrate. Therefore, the volume change used in eqn. 1 is too large and  $\Delta n$  is underestimated. The only direction of free expansion is in the direction normal to the surface and results in a swelling of the glass.<sup>2</sup> An example of such swelling is shown in Fig. 1 as a height discontinuity at the boundary of the exchanged area. The state of stress at the surface is described by

$$\sigma_y = \sigma_z = \sigma_0 \quad \sigma_x = 0 \quad (2)$$

with the  $x$  co-ordinate in the direction normal to the surface. Using Hooke's law, the strains ( $\epsilon_x, \epsilon_y, \epsilon_z$ ) corresponding to

these stresses can be calculated, and yield the following change in volume (from the unstressed state):

$$\frac{\Delta V'}{V} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{2\sigma_0}{E} (1 - 2\nu) \quad (3)$$

where  $E$  is Young's modulus and  $\nu$  the Poisson ratio. For soda-lime glasses, we have<sup>9</sup>

$$E = 7.2 \times 10^4 \text{ N/mm}^2 \quad \nu = 0.21$$

To evaluate eqn. 3, the magnitude of  $\sigma_0$ , the maximum stress at the surface of the substrate, is needed. From References 7 and 8, we can estimate  $\sigma_0$  to lie between  $-700$  and  $-1000$  N/mm<sup>2</sup>. Of course, the exact value depends on a number of factors, such as the glass composition and the temperature of the process. Lower temperatures in particular should result in higher stresses because of the reduced viscosity. Taking the highest value of stress to compare the resulting  $\Delta n$  with our index change data at a relatively low temperature (350–400°C), we obtain from eqn. 3 that

$$\frac{\Delta V'}{V_0} = -0.017 \quad \Delta V' = -0.25$$

Then the net volume change (from ionic change minus the compression) is

$$\Delta V_{net} = \Delta V + \Delta V' = 1.054 - 0.25 = 0.804$$

Substituting in eqn. 1, the new value for  $\Delta V$  yields  $\Delta n_{max} = 0.0089\chi$ , in good agreement with the measured values, which range from 0.008 (at  $T = 440^\circ\text{C}$ ) to 0.009 (at  $360^\circ\text{C}$ ) for TE modes in soda-lime glass waveguides.<sup>4</sup> In addition to that, the stress analysis can also be used to predict the birefringence which has been observed in these waveguides<sup>4</sup> (the index change for TM modes was found to lie between 0.0014 and 0.0021 higher than for their TE counterparts).

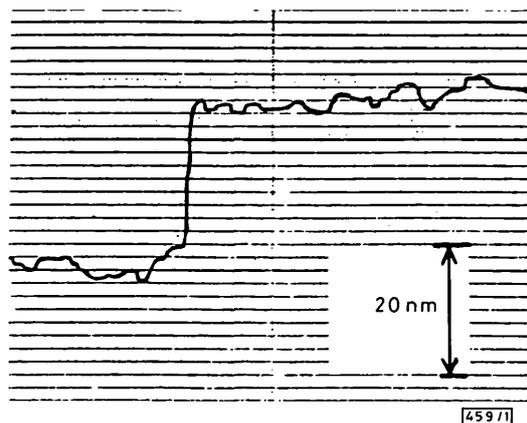


Fig. 1 Surface swelling at boundary of ion-exchanged area  
 $T = 360^\circ\text{C}$ , exchange time = 1 h

The birefringence of anisotropically stressed glass is given by<sup>9</sup>

$$\delta n = \Delta n_{TM} - \Delta n_{TE} = -B\sigma_0 \quad (4)$$

where  $B$  is the birefringence factor of the material. For soda-lime glass<sup>9</sup>  $B \approx 2.4 \times 10^{-6} \text{ mm}^2/\text{N}$ . This gives a  $\delta n$  value of  $2.4 \times 10^{-3}$ , somewhat higher but still close to the measured values. The same calculations were performed for the case of silver-sodium ( $Ag^+-Na^+$ ) exchange, and the results are summarised in Table 1.

Table 1

Ions	$\Delta V$	$\Delta R$	$\Delta n$	$\Delta V'$	$\Delta n(+\text{str.})$	$\Delta n(\text{exp})$	$\delta n$	$\delta n(\text{exp})$
$K^+-Na^+$	1.054	0.546	0.0003	-0.25	0.0089	0.008-0.009	0.0024	0.0014-0.0021
$Ag^+-Na^+$	0.62	1.542	0.082	-0.05	0.083	0.09	0.0005	—

$V_0 = 15 \text{ cm}^3$ ,  $R_0 = (n-1)V_0 = 7.7 \text{ cm}^3$ ; all  $\Delta V$  and  $\Delta R$  in the Table are in  $\text{cm}^3$   
Polarisability in  $\text{\AA}^3$ :  $Na^+ = 0.41$ ,  $K^+ = 1.33$ ,  $Ag^+ = 2.40$

We have shown that the refractive index increase and birefringence resulting from potassium–sodium ion exchange in soda-lime glass are almost exclusively due to a stress-induced surface effect. This is in total contrast with the case of silver–sodium exchange, where the difference in polarisability of the ions dominates both the volume change effect and the correction to it due to the induced stresses in the calculation of  $\Delta n$ . This is clearly shown in Table 1 (the maximum value of stress observed<sup>8</sup> for that type of exchange is  $-200 \text{ N/mm}^2$ ). Also, in that case, no birefringence has been reported. Using our analysis, the birefringence should be about  $5 \times 10^{-4}$ .

These findings will be useful for the design of new structures which involve modifications of the index profile through changes in the fabrication process, as shown in the following example. It is possible to bury a waveguiding layer below the surface of the substrate by making a second ion-exchange with the ions which were originally present in the glass<sup>10</sup> (i.e.  $\text{K}^+ - \text{Na}^+$  followed by  $\text{Na}^+ - \text{K}^+$ ). In principle this would lead to reduced scattering loss due to imperfections at the glass/air interface (e.g. dust or roughness). However, the present analysis shows that the scattering loss may be increased instead of reduced because the second process induces tensile stresses in the originally compressed glass, which may help to promote crack formation and propagation.<sup>7,8</sup> In view of what has been presented in this letter, a more efficient way of burying a waveguide layer would be to subject an ordinary ion-exchanged substrate to a very high temperature (above the softening temperature<sup>9</sup> of  $735^\circ\text{C}$ , for instance) for a short period of time. This would lead to total stress relaxation at the surface by allowing glass flow (thereby locally reducing the index change to zero), while deeper layers would remain unaffected because of the relatively poor thermal conductivity of glass.

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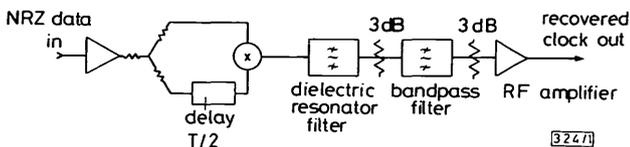
**2 Gbit/s TIMING RECOVERY CIRCUIT USING DIELECTRIC RESONATOR FILTER**

*Indexing terms:* Optical communications, Timing circuits, Filters

Detailed techniques and experimental results are given on circuitry for clock recovery in a 2 Gbit/s digital communications system. The approach used can readily be extended to data rates in excess of 10 Gbit/s.

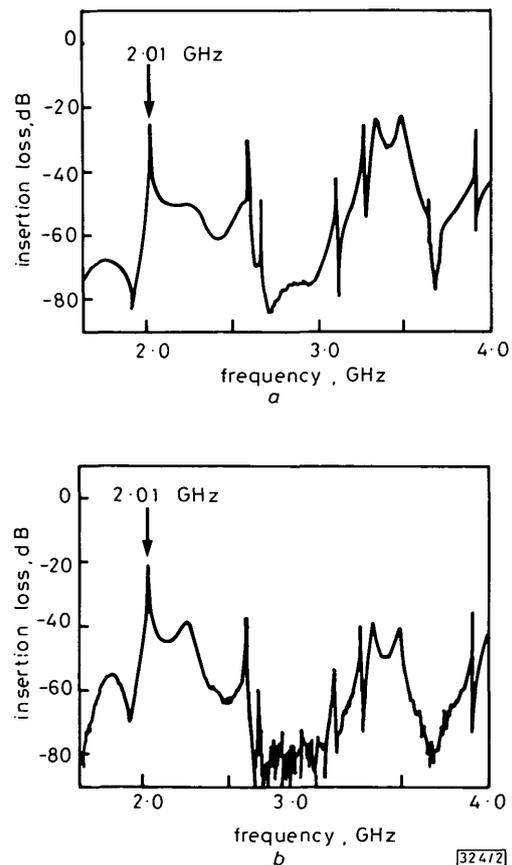
**Introduction:** Advances in fibre optics and semiconductor device technology are making possible digital communications systems operating at Gbit/s data rates. Such systems require electronic circuits capable of performing amplification, retiming and pulse regeneration at comparable rates. In this letter we describe an experimental 2 Gbit/s timing recovery circuit for the NRZ transmission format. We have previously reported on a 432 Mbit/s timing recovery circuit using an SAW filter for clock extraction.<sup>1</sup> In the present letter we show how those techniques can be extended to permit timing recovery at transmission rates of 2 Gbit/s using higher-frequency components, and substituting a dielectric resonator filter (DRF) for the SAW filter. Advantages of this approach include improved jitter performance, low insertion loss and simple construction. In addition, the techniques developed in this approach can be extended to data rates in excess of 10 Gbit/s.

**Circuit description:** The 2 Gbit/s timing recovery circuit is shown schematically in Fig. 1. The NRZ data stream is amplified and split into two paths using a resistive splitter. The first



**Fig. 1** 2 Gbit/s timing recovery circuit

path is fed into the LO port of a double balanced mixer, while the second path is delayed by half a bit period and fed into the RF port. The mixing action causes a relatively strong spectral component to be generated at the data rate. This technique is



**Fig. 2** Dielectric resonator filter characteristics  
a Without bandpass filter  
b With bandpass filter