Resolution guarantees in EIT including random and systematic errors

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Abstract: To improve the practical applicability of electrical impedance tomography is a great ongoing challenge. Theoretical identifiability results exist for noiseless continuous boundary measurements. However, little is known about what can be achieved with a finite number of realistically modelled electrodes in a setting including modelling and measurement errors. In this paper, we sketch how to derive rigorous resolution guarantees for such settings.

1 Introduction

Notation: χ_M denotes the characteristic function of a set M and $\mathrm{eig}(A)$ the set of eigenvalues of a square matrix A.

We consider a conductive object $\Omega \subseteq \mathbb{R}^n$ ($n \in \{2,3\}$) with a conductivity distribution

$$\sigma : \Omega \to \mathbb{R}, \quad \sigma(x) = \sigma_B(x)\chi_{\Omega\setminus D}(x) + \sigma_D(x)\chi_D(x), \quad (1)$$

where $\sigma_B(x)$ is the background conductivity and $\sigma_D(x)$ the inclusion conductivity of an inclusion $D \subseteq \Omega$. The inclusion is characterized by a contrast to the background with

$$\inf_{x\in D} \sigma_D(x) \geq \sigma_{D\min} > \sup_{y\in\Omega\setminus D} \sigma_B(y), \quad \sigma_{D\min} \in \mathbb{R}. \quad (2)$$

Furthermore, let $(\omega_1, \omega_2, \cdots, \omega_N)$ be a resolution partition of Ω (see Figure 1). In Section 3, we sketch how to verify if a realistically modelled measurement setting (see Section 2) yields enough information to design an inclusion detection method that fulfils the following guarantee.

Resolution guarantee (RG):

(a) A resolution element ω_i will be marked if $\omega_i \subseteq D$.

(b) No resolution element will be marked if $D = \emptyset$.

2 The measurement setting

The setting is given by current-voltage measurements on a finite number of (almost perfectly conductive) electrodes $E_1, E_2, \ldots, E_L$. We assume that a contact layer between each electrode E_i and Ω leads to a contact impedance $z^{[i]}$.This setting is mathematically modelled by the complete electrode model (CEM), cf. [1]. For a conductivity distribution σ and contact impedances given by the components of $z \in \mathbb{R}^L$, the measurement matrix is defined by

$$R(\sigma, z) = \left(R^{[i,j]}(\sigma, z)\right)_{i,j=1}^{L-1} \in \mathbb{R}^{L-1\times L-1}, \quad (3)$$

where the components $R^{[i,j]}(\sigma, z)$ are given by the measurements as in Fig. 1. The matrix $R(\sigma, z)$ is symmetric, cf. [1].

To allow for **modelling and measurement errors**:

(a) The background conductivity $\sigma_B(x)$ is given approximately by $\sigma_0(x)$ with $\|\sigma_B - \sigma_0\|_\infty \leq \varepsilon \in \mathbb{R}$.

(b) The vector z (contact impedances) is given approximately by z_0 with $\|z - z_0\|_\infty \leq \gamma \in \mathbb{R}$.

(c) There are noisy measurements $R_\delta(\sigma, z)$ given with an absolute noise level $\delta \geq \|R(\sigma, z) - R_\delta(\sigma, z)\|_2$, $\delta \in \mathbb{R}$. Possibly replacing $R_\delta(\sigma, z)$ by its symmetric part, we can assume that $R_\delta(\sigma, z)$ is symmetric.

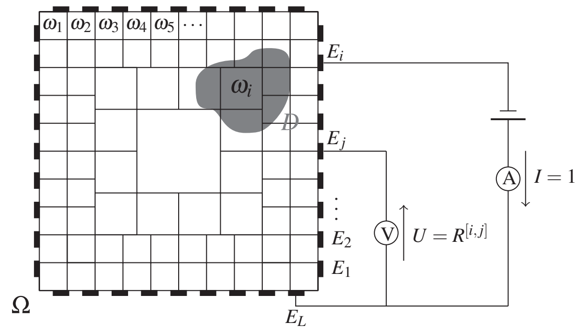

Figure 1: Setting with a sample resolution for $\Omega = [-1, 1]^2$.

3 Verification of the resolution guarantee

Let $\sigma_0(x)$, z_0, ε, γ, δ and $\sigma_{D\min}$ be given. We define

$$\sigma_{B\min}(x) := \sigma_0(x) - \varepsilon, \quad \sigma_{B\max}(x) := \sigma_0(x) + \varepsilon, \quad (4)$$

$$z_{\min} := z_0 - \gamma(1, \ldots, 1), \quad z_{\max} := z_0 + \gamma(1, \ldots, 1), \quad (5)$$

$$\tau_i(x) := \sigma_{B\min}(x)\chi_{\Omega\setminus\omega_i}(x) + \sigma_{D\min}\chi_{\omega_i}(x) \quad (6)$$

for $i \in \{1, 2, \ldots, N\}$. Then the **RG is possible** if

$$\max_{i=1}^{N} \min \mathrm{eig}\left(R(\tau_i, z_{\max}) - R(\sigma_{B\max}, z_{\min})\right) < -2\delta. \quad (7)$$

The proof is based on the monotonicity relation

$$\sigma_1 \leq \sigma_2,\ z_1 \geq z_2 \quad \Rightarrow \quad R(\sigma_1, z_1) - R(\sigma_2, z_2) \geq 0. \quad (8)$$

The main idea is to consider (7) as a worst-case scenario test for Algorithm 1 (cf. [2] for $\gamma = 0$).

Algorithm 1: Mark element ω_i if

$$\min \mathrm{eig}\left(R(\tau_i, z_{\max}) - R_\delta(\sigma, z)\right) \geq -\delta. \quad (9)$$

3.1 Numerical results

Let Ω be given with a resolution partition as in Fig. 1. Furthermore, let $\sigma_0 \equiv 1$ and $z_0 = (1, \ldots, 1) \in \mathbb{R}^L$ be approximations of the background conductivity $\sigma_B(x)$ and the vector z (contact impedances), respectively. Additionally, let $\sigma_{D\min} = 2$ be a lower bound of the inclusion conductivity.

Then (7) is fulfilled for a background error of $\varepsilon = 1\%$, an absolute measurement noise level of $\delta = 0.9\%$ and exactly given contact impedances ($\gamma = 0$). Hence, the RG is possible. In particular, Algorithm 1 fulfils the RG.

The results can be extended to the case of approximately known contact impedances.

4 Conclusion

This paper presents the possibility of a rigorous resolution guarantee for a realistically modelled electrode measurement setting including modelling and measurement errors. The resolution guarantee can be verified by a simple test.

References

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Excerpted from:

Proceedings of the 15th International Conference on Biomedical Applications of

ELECTRICAL IMPEDANCE TOMOGRAPHY

Edited by Andy Adler and Bartłomiej Grychtol

April 24-26, 2014
Glen House Resort
Gananoque, Ontario
Canada

This document is the collection of papers accepted for presentation at the 15th International Conference on Biomedical Applications of Electrical Impedance Tomography.

Cover design: Bartłomiej Grychtol
Photo credit: ©1000 Islands Photo Art Inc. / Ian Coristine

Printed in Canada

ISBN 978-0-7709-0577-4

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