CEP 14-11

Horizontal Mergers in the Presence of Capacity Constraints

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September 2014

CARLETON ECONOMIC PAPERS

Department of Economics

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Ottawa, Ontario, Canada
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by

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August 2014

Abstract

We analyze the effects of a merger between two competitors in a Bertrand-Edgeworth model. The merger has no effect on equilibrium prices if a pure strategy equilibrium prevails both before and after the merger. Otherwise, the merger leads to higher prices. In the case where a mixed strategy equilibrium prevails before and after the merger, for example, the support of the price distributions shifts rightward after the merger and the post-merger price distribution of each firm stochastically dominates its pre-merger counterpart. The pre-merger capacity level of each firm plays a crucial role in determining the effects of the merger.

Key words: Merger, capacity constraints, Bertrand-Edgeworth model

JEL Classification: L13, L40

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We would like to thank Gamal Atallah, Yongmin Chen, Thomas Ross, Aggey Semenov and Guofu Tan for their comments. We also benefited from the discussions with seminar participants at Shanghai Jiaotong University, Shanghai University of Finance and Economics, and Shangdong University.

1. Introduction

A commonly held view among economists is that, in the absence of efficiency gains, horizontal mergers between oligopolistic firms raise prices.[1] But recent works by Froeb, *et al.* (2003) and Higgins, *et al.* (2004) challenge this view. In particular, Froeb, *et al.* (2003) conduct numerical simulations of a merger model with price competition among capacity-constrained firms, and they find that "[i]n the case where the merged firm is capacity-constrained, there is no merger price effect." To be more specific, they show that, although the capacity constraints on the non-merging firms drive up post-merger prices as has long been believed, the capacity constraints on the merging firms depress post-merger prices, and the latter effect is greater than the former. When both merging firms are capacity constrained, a merger has simply zero effect on price, quantity, consumer surplus and total welfare. Extending the approach of Froeb, *et al*. (2003), Higgins, *et al.* (2004) reaffirms this finding in a more elaborate computation model.

This finding has siginficant implications for competition policy towards mergers. If a merger has no price effect whenever the merging firms are capacity constrained, this would provide antitrust authorities with a simple test to screen out a class of mergers that are not expected to cause any harm to competition. This test would be very useful because capacity constraints are ubiquitous in reality. For example, the output that a manufacturer can produce during any given period is constrained by the size of its plants, and the maximum number of guests that a hotel can accommodate in a given night is determined by the number and capacity of rooms it has. Indeed, it can be argued that in the short run firms in every industry face capacity constraints; the difference is that the capacity constraints may be tighter in some

[1] For theoretical demonstrations of this point, see, for example, Salant, *et al*. (1983), Deneckere and Davidson (1985), McAfee, *et al*. (1992), Perry and Porter (1985), Farrell and Shapiro (1990). Empirically, Weinberg (2008) surveys the literature on the price effects of horizontal mergers, and finds that prices increased after a merger in the majority of cases that have been examined.

industries than in others. However, before this finding of Froeb, *et al*. (2003) can be applied with confidence to merger enforcement, its robustness must be assessed.

The objective of this paper is to investigate the price effects of merger in the presence of capacity constraints using an analytical model (as opposed to numerical simulations). Specifically, we analyze a merger between two competitors in the familiar Bertrand-Edgeworth model, where n symmetric firms produce a homogeneous product and they set prices simultaneously taking into consideration their capacity constraints.

Our analysis identifies the conditions under which a merger does or does not have price effects in the Bertrand-Edgeworth model. Specifically, we show that the characteristics of the equilibriums and accordingly the effects of the merger depend on the pre-merger capacity level of each firm. If the capacity level is either very low or very high, firms play pure strategies in equilibrium both before and after the merger. In this case, the merger has no effect on the equilibrium prices, as in Froeb, *et al.* (2003). If the capacity level is in the intermediate range, however, a mixed strategy equilibrium occurs and higher prices prevail after the merger. To be more specific, in the case where the merger turns a pure strategy equilibrium into a mixed strategy equilibrium, the range of post-merger prices is strictly higher than the pre-merger prices. In the case where a mixed strategy equilibrium occurs both before and after the merger, moreover, the support of the price distributions shifts rightward after the merger and the post-merger price distribution of each firm stochastically dominates its pre-merger counterpart. Consequently, the merger enables every firm (including the merged entity) to earn a larger profit whenever a mixed strategy equilibrium prevails after the merger.

Therefore, our analysis demonstrates a number of possible scenarios for a merger in markets where firms face capacity constraints. For the merger to have no price effect, it is not sufficient that the merged firm faces a capacity constraint; this capacity constraint and the counterparts of all other firms have to be binding both before and after the merger. Binding capacity constraints before the merger does not necessarily mean that they will remain binding after the merger. The merger can turn a binding capacity constraint into a slack one. On the other hand, in situations where the capacity is so large that all firms charge a price equaling marginal cost in the pre-merger equilibrium, the merger (which merely combines the capacity of two firms without changing the aggregate level of capacity) can cause the equilibrium price to rise above the marginal cost. The implication of these findings is that in industries where capacity constraints are a significant factor, the tightness of these constraints plays an important role in determining the effects of a merger.

There is a sizeable literature on the effects of horizontal mergers in Cournot and Bertrand models of oligopoly. It is now well known that in the absence of efficiency gains, mergers under Cournot competition are often unprofitable (*i.e.*, the "merger paradox").[2] On the other hand, a number of authors (Deneckere and Davidson 1985, Braid 1986 and 1999, Reitzes and Levy 1990, and Levy and Reitzes 1992) have shown that a merger raises prices and is more profitable for the merging firms when firms produce differentiated products and compete in prices (*i.e.*, under Bertrand competition).

Surprisingly, very few authors have studied the effects of merger in the Bertrand-Edgeworth model despite its prominence in the oligopoly theory.[3] Our literature search has uncovered only two such studies, namely Davidson and Deneckere (1984) and Hirata (2009). Davidson and

[2] See, for example, Salant, *et al*. (1983), and Lommerud and Sørgard (1997). A number of authors, including Perry and Porter (1985), McAfee, *et al.* (1992) and Daughety (1990), have studied various ways to resolve the merger paradox.

[3] Notable analyses of the Bertrand-Edgeworth model include Kreps and Scheinkman (1983), Allen and Hellwig (1986), Dasgupta and Maskin (1986a and 1986b), Osborne and Pitchik (1986) and Vives (1986) . More recently, Hirata (2009) and De Francesco and Salvadori (2009 and 2010) investigate the case of asymmetric firms in more general settings.

Deneckere (1984) study the impact of merger on tacit collusion in a Cournot model and a Bertrand-Edgeworth model with a linear demand function. Hirata (2009) contains a brief analysis on the possibility that a merger could be unprofitable when firms face capacity constraints. Neither of them, however, directly studies the price effect of merger when firms act independently (*i.e.*, the unilateral effect of merger).

The remainder of this paper is organized as follows. A Bertrand-Edgeworth model with n symmetric firms is presented in section 2, while the equilibrium in this model is analyzed in section 3. The effects of a merger between two firms are investigated in section 4. Section 5 concludes.

2. Bertrand-Edgeworth Model

Consider an industry with $n\ (\geq 3)$ symmetric firms that produce a homogeneous good. The marginal cost of production is normalized to zero and there is no fixed cost. Each firm has a production capacity of K; in other words, the output of each firm cannot exceed K. Firms compete by setting their prices simultaneously.

The market demand for the good is represented by the function $Q = D(p)$. Then the demand at the price equal to marginal cost is $D(0)$, which is assumed to be a finite number. As is common in the literature on the Bertrand-Edgeworth model (*e.g.*, Vives 1986, Hirata 2009, De Francesco and Salvadori 2009), we assume that $D(\cdot)$ is decreasing, concave and twice continuously differentiable. These assumptions imply that $pD(\cdot)$ is strictly concave in p.

Let $P(Q) = D^{-1}(Q)$ denote the corresponding inverse demand function. The assumptions on $D(\cdot)$ then imply that the inverse demand function satisfies $P'(Q) < 0$ and $P''(Q) \leq 0$.

In this model, firms may charge different prices. Consumers will want to buy from whichever firm that offers the lowest price. However, when the demand for a firm's product exceeds its capacity, consumers have to be rationed. We follow the common approach in the literature, and assume the efficient rationing rule under which the output of the lowest-price firms is sold first to consumers with the highest valuation.[4] Accordingly, if firm j offers the lowest price p_j but cannot satisfy $D(p_j)$ due to its capacity constraint, the residual demand facing the firm offering the second lowest price p_i is given by $D(p_i) - K_j$. In the symmetric case where all n firms have the same capacity K, a firm charging the hth lowest price faces the residual demand of $\max\{0, D(p_h) - (h-1)K\}$. Moreover, if several firms charge the same price, the residual demand is shared among the equally-priced firms in proportion to their capacities.

An important element in the analysis of the Bertrand-Edgeworth model is the Cournot best-response function. Define $q_i\left(\sum_{j\neq i} K_j\right)$ as firm i's Cournot best-response function when all of its rivals produce at their capacity levels. In other words, $q_i\left(\sum_{j\neq i} K_j\right)$ is solved from firm i's first-order condition

$$P\left(q_i + \sum_{j\neq i} K_j\right) + q_i P'\left(q_i + \sum_{j\neq i} K_j\right) = 0. \qquad (1)$$

By definition, $q_i\left(\sum_{j\neq i} K_j\right)$ represents firm i's profit-maximizing output level when all other firms produce at full capacity. This implies that, under price competition, firm i will have no incentive to undercut its rivals' prices if its capacity is smaller than $q_i\left(\sum_{j\neq i} K_j\right)$.

From (1), it is easy to derive the slope of the firm's Cournot best-response function:

$$\frac{dq_i}{d\sum_{j\neq i} K_j} = -\frac{P' + q_i P''}{2P' + q_i P''}\,. \qquad (2)$$

[4] The efficient rationing rule is used in Levitan and Shubik (1972), Kreps and Scheinkman (1983), Davidson and Deneckere (1984), Osborne and Pitchik (1986), Vives (1986), Hirata (2009), and De Francesco and Salvadori (2009).

Using the properties of the inverse demand function, we can verify that the value of (2) falls in the interval $(-1, 0)$. In other words, the Cournot best-response curve is downward-sloping with (the absolute value of) the slope less than 1.

3. Pre-Merger Equilibrium

Our objective is to investigate the effects of a merger between two firms. We will achieve this by comparing the equilibriums before and after the merger. Before the merger, there are n symmetric firms, each with a capacity K. The equilibrium in such a situation has been studied by Vives (1986). Below we reproduce some of his results that are relevant for our later analysis.

It is well-known in this literature that depending on the firms' capacity levels, an equilibrium in the Bertrand-Edgeworth model may involve mixed strategies. We will start by characterizing the critical levels of capacity K that separate a mixed strategy equilibrium and a pure strategy equilibrium.

It is shown in Vives (1986) that there are two such critical levels of K. The first one is $D(0)/(n-1)$, where $D(0)$ is the quantity demanded at the price equal to marginal cost. If the capacity level of each firm is so large that $K \geq D(0)/(n-1)$, each firm chooses $p = 0$ and earns zero profit in equilibrium. In other words, we have the classic Bertrand equilibrium where the price equals marginal cost.

To introduce the second critical level of K, note that the solution to (1), the Cournot best-response function, can now be written in the form $q_i((n-1)K)$. Let $\widetilde{K}$ denote the solution to $\widetilde{K} = q_i((n-1)\widetilde{K})$. It is proven by Vives (1986) that there is a pure strategy equilibrium if the capacity of each firm is so small that $K \leq \widetilde{K}$. In this equilibrium, every firm charges the price $P(nK)$, produces at full capacity K, and earns a profit $KP(nK)$.

A mixed strategy equilibrium occurs if K is in the intermediate range, *i.e.* if $\widetilde{K} < K < D(0)/(n-1)$. Let $F(p)$ denote the equilibrium mixed strategy of a firm; that is, $F(p) = \Pr(p_i \leq p)$ is the probability of firm i charging a price lower than p. To put it differently, $F(p)$ is the cumulative distribution function of the firm's equilibrium prices. Let $[p^l, p^h]$ be the support of $F(p)$ and π the firm's expected profit in equilibrium. Then Proposition 1 in Vives (1986) implies that

$$p^h = \operatorname{argmax}_p\, p[D(p) - (n-1)K], \qquad p^l = \frac{p^h[D(p^h) - (n-1)K]}{K}. \tag{3}$$

$$F(p) = \left[\frac{(p - p^l)K}{p[nK - D(p)]}\right]^{1/(n-1)}, \qquad \pi = p^l K = p^h[D(p^h) - (n-1)K]. \tag{4}$$

From (3) we can verify that $P(nK) < p^l < p^h < P((n-1)K)$. That is, the support of equilibrium price distribution is bounded by $P(nK)$ and $P((n-1)K)$. Moreover, it can be shown that as n increases, p^l and p^h fall and thus the range of equilibrium prices (p^l, p^h) shifts toward the left. Intuitively, an increase in the number of firms (and a larger total production capacity) intensifies competition, thus reducing the price and the profit of every firm. Similarly, an increase in K reduces both p^l and p^h and causes the range of equilibrium prices to shift leftward. This is not surprising since firms compete more aggressively when they have larger production capacity.

For ease of later discussion, we summarize the pre-merger equilibrium as follows.

Lemma 1: Before the merger,

(i) a pure strategy equilibrium where each firm sets $p_i = P(nK)$ prevails if $K \leq \widetilde{K}$.

(ii) a mixed strategy equilibrium prevails if $\widetilde{K} < K < D(0)/(n-1)$.

(iii) a pure strategy equilibrium where each firm sets $p_i = 0$ prevails if $K \geq D(0)/(n-1)$.

The proofs of this and all remaining lemmas and propositions are relegated to the appendix.

4. Effects of Merger

Suppose firms 1 and 2 merge to form a new entity M while the other firms remain unchanged. The merger leads to an oligopoly market where one firm has a capacity twice as large as that of a remaining firm. This means that firms are no longer symmetric.

This asymmetry creates challenges for the equilibrium analysis because it is generally difficult to obtain a complete characterization of mixed strategy equilibriums in the Bertrand-Edgeworth model with more than three asymmetric firms (De Francesco and Salvadori 2009). Fortunately, our model involves a very special situation where only one firm has the largest capacity and all remaining firms have the same capacity. To further simplify analysis, we focus on equilibriums in which firms with the same capacity choose the same strategy. In the case of a mixed strategy equilibrium, for example, all non-merging firms (*i.e.*, firms 3 through *n*) adopt the same price distribution.

Despite these simplifications, the derivations involving mixed strategy equilibriums are still quite long and tedious. For this reason, we relegate most of the technical details to the appendix.

For ease of discussion, we will follow the convention of referring to firms 1 and 2 as the "insiders" and the remaining firms as the "outsiders". Note that the combined capacity of all outsiders is $(n-2)K$. Thus, the Cournot best-response function of the merged firm M is $q_M\big((n-2)K\big)$. Let K_M denote the capacity of firm M. Then $K_M = 2K$.

Lemma 2: There exists a unique $\widetilde{K}_M$ such that $\widetilde{K}_M = q_M((n-2)\widetilde{K}_M/2)$, and that $K_M \leq q_M((n-2)K_M/2)$ if and only if $K_M \leq \widetilde{K}_M$.

Lemma 2 determines a critical level of firm M's capacity above which its capacity exceeds its Cournot best-response output when all outsiders produce at full capacity. This, in turn, helps define one of the two thresholds of K that separate pure strategy equilibriums and mixed strategy equilibriums. The other threshold is $D(0)/(n-2)$.

Lemma 3: After the merger,

(i) a pure strategy equilibrium where each firm sets $p_i = P(nK)$ prevails if $K \leq \widetilde{K}_M/2$.

(ii) a mixed strategy equilibrium prevails if $\widetilde{K}_M/2 < K < D(0)/(n-2)$.

(iii) a pure strategy equilibrium where each firm sets $p_i = 0$ prevails if $K \geq D(0)/(n-2)$.

From Lemma 3, we see that qualitatively the post-merger equilibriums are the same as the pre-merger equilibriums. Specifically, a pure strategy equilibrium prevails if K is either very large or very small, but a mixed strategy equilibrium occurs if K is in the intermediate range.

For the purpose of this analysis, it is important to note that the critical values of K that separate the pure strategy equilibriums from the mixed strategy equilibriums are different before and after the merger. It is easy to see that $D(0)/(n-2) > D(0)/(n-1)$. This implies that if K falls between $D(0)/(n-1)$ and $D(0)/(n-2)$, the merger turns a pure strategy equilibrium into a mixed strategy equilibrium. On the other hand, a careful comparison of $\widetilde{K}_M$ and $\widetilde{K}$ reveals the following.

Lemma 4: $\widetilde{K}_M < 2\widetilde{K}$.

Lemmas 3 and 4 then imply that the merger reduces the equilibrium range of K for which every firm sets $p_i = P(nK)$ and produces at its full capacity, from $(0, \widetilde{K})$ to $(0, \widetilde{K}_M/2)$. Intuitively, this arises because the merger decreases the insiders' (combined) Cournot best-response output from $2\widetilde{K}$ to $\widetilde{K}_M$. It is well-known in the literature on the Bertrand-Edgeworth model that, in the case where firms have different levels of capacity, the occurrence of the pure strategy equilibrium is determined by the firm with the largest capacity (De Francesco and Salvadori 2009). This pure strategy equilibrium would arise as long as the firm's capacity is below its Cournot best-response output when all other firms produce at full capacity. In our model, firm M has the largest capacity after the merger. Lemma 4 implies that the Cournot best-response output of firm M does not increase by the same proportion as its capacity. This reduces the critical value of K under which the pure strategy equilibrium occurs.

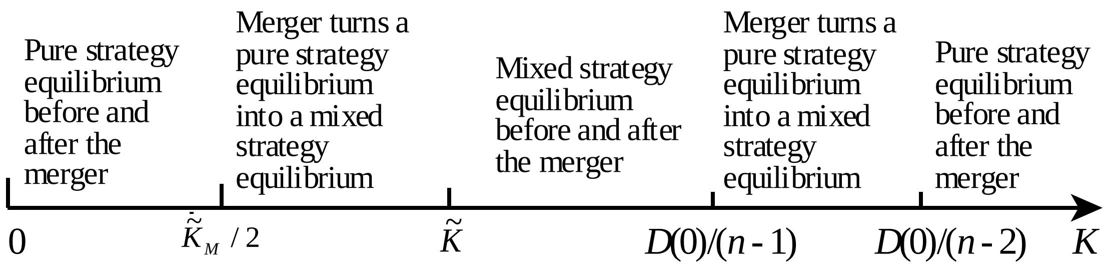

Figure 1: Critical Values of K

Lemmas 1–4 suggest that, in terms of the characteristics of the pre- and post-merger equilibriums, there are a total of five different cases. As shown in Figure 1, a pure strategy equilibrium prevails before and after the merger if $K \leq \widetilde{K}_M/2$. The same is true if $K \geq D(0)/(n-2)$. A pure strategy equilibrium before the merger becomes a mixed strategy equilibrium after the merger if $\widetilde{K}_M/2 < K \leq \widetilde{K}$, or if $D(0)/(n-1) \leq K < D(0)/(n-2)$. Finally, a mixed strategy equilibrium prevails before and after the merger if $\widetilde{K} < K < D(0)/(n-1)$.

Next, we investigate how the merger affects the equilibrium prices. We can divide the five cases in Figure 1 into two sets. In the first set, a pure strategy equilibrium prevails both before and after the merger. In the second set, a mixed strategy equilibrium occurs after the merger. It turns out the effects of the merger are very different for these two sets of cases.

Proposition 1: If $K \leq \widetilde{K}_M/2$ or $K \geq D(0)/(n-2)$, a pure strategy equilibrium prevails before and after the merger, and the merger has no effect on equilibrium prices.

As stated in Proposition 1, the first set of cases arise when the capacity level is either sufficiently large ($K \geq D(0)/(n-2)$) or sufficiently small ($K \leq \widetilde{K}_M/2$). The case of large capacity is not very interesting. It involves a situation where the capacity levels of all firms are so large that the classic Bertrand equilibrium (where the price equals marginal cost) prevails before and after the merger. This can be seen mostly clearly when there are three firms before the merger ($n = 3$). Here a sufficiently large capacity means that $K \geq D(0)$, *i.e.*, every firm has enough capacity to serve the entire market. Thus, we have the standard Bertrand competition among firms that drives the price down to the marginal cost both before and after the merger.

In the case where $K \leq \widetilde{K}_M/2$, the capacity level of every firm is so small that capacity constraint of every firm is binding both before and after the merger. Consequently, each firm produces at full capacity and the equilibrium price is pinned down at $P(nK)$, which is above the marginal cost. Since the merger does not change the total capacity of all firms (nK), it has no effect on equilibrium prices.

The equilibrium in this case of small capacity is consistent with the finding by Froeb, *et al*. (2003 p59) that "[i]n the case where the merged firm is capacity-constrained, there is no merger price effect." It should be emphasized that for a merger to have no price effect, the capacity constraints must be binding both before and after the merger. The latter, however, does not always hold. As will be elaborated below, there are instances where the capacity constraints are binding before the merger but they become slack after the merger.[5]

In the second set of cases, the post-merger equilibriums involve mixed strategies. This occurs if $\widetilde{K}_M/2 < K < D(0)/(n-2)$. Let $F_M(p)$ and $F_O(p)$ denote a mixed strategy of firm M and each outsider, respectively. It can be shown that these two distributions have the same support in equilibrium. Let p^H and p^L denote the supremum and infimum of the equilibrium price distribution. Then in equilibrium, $F_M(p)$ and $F_O(p)$ are continuous and increasing functions over the range (p^L, p^H).

Using the standard argument of the Bertrand-Edgeworth model, we obtain the supremum and infimum of the price support as follows:

$$p^H = \text{argmax}_p\, p[D(p) - (n-2)K], \tag{5}$$

$$p^L = \frac{p^H[D(p^H) - (n-2)K]}{\min\{2K,\ D(p^L)\}}. \tag{6}$$

Conditions (5) and (6) imply that $P(nK) < p^L < p^H < P((n-2)K)$. In other words, in a mixed strategy equilibrium after the merger, the equilibrium prices will not go below $P(nK)$ or above $P((n-2)K)$. Note that $P(nK)$ is the price in a pure strategy equilibrium where every firm produces at full capacity, while $P((n-2)K)$ would be the price if all outsiders produce at full capacity but firm M produces nothing.

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[5] This is in contrary to the belief of Froeb, *et al.* (2003 p59), who state, “If firms are capacity-constrained, they are pricing where potential demand equals capacity. This pricing calculus is less likely to be changed by merger.” Our Proposition 2 below shows that the merger does change the pricing calculus because it alters the tightness of the capacity constraints.

The second set consists of three cases. First, in the case where $\widetilde{K}_M/2 < K \leq \widetilde{K}$, the merger turns a pure strategy equilibrium into a mixed strategy equilibrium. Since the equilibrium price before the merger is $P(nK)$, it follows from $p^L > P(nK)$ that the merger leads to higher prices.

Proposition 2: If $\widetilde{K}_M/2 < K \leq \widetilde{K}$, a pure strategy equilibrium before the merger changes to a mixed strategy equilibrium after the merger where the range of equilibrium prices are strictly higher than the pre-merger price $P(nK)$.

One interesting observation from Proposition 2 is that in the case where the capacity constraints are binding (*i.e.*, all firms produce at full capacity) in the pre-merger equilibrium, the merger can relax the tightness of the constraints. Since $p^L > P(nK)$, the total quantity produced by all firms in the mixed strategy equilibrium after the merger is less than nK. In other words, the quantities of firm M and the outsiders are distributed over a range below their respective capacity levels. This, in turn, allows them to charge higher prices after the merger.

Second, in the case where $D(0)/(n-1) \leq K < D(0)/(n-2)$, the merger also turns a pure strategy equilibrium before the merger into a mixed strategy equilibrium after the merger. The difference is that the capacity levels of the firms are so large that the price is drive down to the marginal cost in the pre-merger equilibrium. The merger raises the equilibrium prices in the sense that the range of post-merger prices is strictly above the marginal cost.

Proposition 3: If $D(0)/(n-1) \leq K < D(0)/(n-2)$, the Bertrand equilibrium before the merger changes to a mixed strategy equilibrium after the merger where the range of equilibrium prices are strictly above the marginal cost.

Proposition 3 is interesting in that the merger merely combines the capacity of two firms without affecting the aggregate level of capacity in this market. Yet while this capacity level is large enough to drive the pre-merger equilibrium price to the marginal cost, the combination of capacity by the two firms changes the characteristics of the equilibrium and causes the prices of all firms to rise.

Third, if K is in the intermediate range where $\widetilde{K} < K < D(0)/(n-1)$, a mixed strategy equilibrium prevails before and after the merger. Compared with the two cases discussed above, assessing the price effects of the merger in this case is much more difficult. Nevertheless, we are able to obtain the following results.

Proposition 4: If $\widetilde{K} < K < D(0)/(n-1)$, a mixed strategy equilibrium prevails before and after the merger. Moreover,

(i) The range of equilibrium prices shifts toward the right after the merger, that is, $p^H > p^h$ and $p^L > p^l$.

(ii) The post-merger price distributions F_M and F_O strictly dominate pre-merger price distribution F stochastically at first order. In other words, every firm has strictly greater probability to sell below any particular price (except at p^H) before the merger than after the merger.

Proposition 4 suggests that the price increase caused by the merger in this case can be viewed from two perspectives. First, the equilibrium prices are randomized over a range of higher values after the merger. Second, both the insiders and outsiders are more likely to charge higher prices after the merger.

The impact of the merger on profits can be determined by using Propositions 1–4. It is obvious that the merger has no impact on a firm's profit if it does not cause any change in equilibrium prices, *i.e.*, if $K \leq \widetilde{K}_M/2$ or $K \geq D(0)/(n-2)$. If the merger changes the prices (as in Propositions 2–4), the higher prices tend to raise the profits of firm M and each outside firms. But as a firm charges a higher price, it may sell a smaller quantity. This effect is particularly acute for firm M because, as the firm with the largest capacity, it tends to be undercut by the other firms. Nevertheless, we are able to show that the merger is profitable for all firms including firm M.

Proposition 5: The merger raises the profits of both the insiders and outsiders whenever it has a price effect, *i.e.* whenever K is in the range $(\widetilde{K}_M/2\,, D(0)/(n-2))$.

Therefore, the effects of the merger on prices and profits depend on the pre-merger capacity level of each firm. If the capacity level is either very low or very high, the merger has no impact on prices or profits. For the capacity level in the intermediate range, the merger causes higher prices and improves the profitability of both the insiders and outsiders.

5. Concluding Remarks

We have examined the effects of a merger between two firms in a Bertrand-Edgeworth model. We have shown that the merger has no effect on prices or profits if the production capacity of each firm is either sufficiently small or sufficiently large that a pure strategy equilibrium prevails both before and after the merger. If the capacity is in the intermediate range, however, a mixed strategy equilibrium occurs after the merger, in which case the merger raises the prices and improves the profitability of all firms. In the case where a mixed strategy equilibrium prevails both before and after the merger, the post-merger price distribution of each firm stochastically dominates its pre-merger counterpart.

A couple of assumptions in our model are particularly worth noting. First, we have assumed that all firms have the same capacity before the merger. This has simplified our analysis by substantially reducing the number of possible cases we have to consider. At the same time, it probably has eliminated some forces that may be at play when firms are asymmetric. Second, we have used a standard Bertrand-Edgeworth model where all firms produce a homogeneous product. Allowing product differentiation will likely influence the effects of a merger. Deneckere and Davidson (1985) have shown that a merger leads to higher prices when firms produce differentiated products. It will be interesting to investigate whether the presence of capacity constraints strengthens or mitigates this price effect.

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Appendix

Proof of Lemma 1: It follows from Proposition 1 in Vives (1986). ■

Proof of Lemma 2: Firm M's Cournot best-response function $q_M((n-2)K)$ is the solution to $\max_q qP(q+(n-2)K)$. Using (2), we obtain

$$\frac{dq_M}{dK} = -(n-2)\frac{P' + qP''}{2P' + qP''} < 0. \qquad (A1)$$

Given the assumptions on the demand function, we have $q_M(0) > 0$ and $q_M((n-2)K) = 0$ at $K = D(0)/(n-2)$. The continuity and monotonicity of $q_M(\cdot)$ imply that there is a unique $\widetilde{K}_M$ such that $\widetilde{K}_M = q_M((n-2)\widetilde{K}_M/2)$. Since the function $K_M - q_M((n-2)K_M/2)$ is increasing in K_M, we conclude that $K_M \leq q_M((n-2)K_M/2)$ if and only if $K_M \leq \widetilde{K}_M$. ■

Proof of Lemma 3: It follows from Proposition 1 in De Francesco and Salvadori (2009) and Lemma 2. ■

Proof of Lemma 4: From (2) we know that the slope of the Cournot best-response function $q_i(\sum_{j\neq i} K_j)$ is strictly between -1 and 0. Then using the Taylor series we obtain:

$$q_i\left((n-2)\widetilde{K}\right) = q_i\left((n-1)\widetilde{K}\right) + \frac{dq_i(\cdot)}{d\sum_{j\neq i} K_j}\left(-\widetilde{K}\right) < 2\widetilde{K}. \qquad (A2)$$

Since the function $2K - q_i((n-2)K)$ is increasing in K, (A2) and the definition of $\widetilde{K}_M$ then imply $\widetilde{K}_M < 2\widetilde{K}$. ■

Proof of Proposition 1: It follows from comparing Lemma 1 and Lemma 3 using Lemma 4. ■

Derivation of p^H and p^L in (5) and (6): The value of p^H is equal to the profit-maximizing price of firm M (the firm with the largest capacity) when it allows its rivals to undercut and produce at their capacities ($(n-2)K$). Hence we have (5).

To derive (6), note that firm M may or may not be capacity-constrained at $p = p^L$; that is, $2K$ may be less than or greater than $D(p^L)$. This ambiguity exists when $n = 3$. But in the case where $n \geq 4$, $(n-2)K \geq 2K$ implies $P((n-2)K) \leq P(2K)$. Then $p^L < p^H < P((n-2)K)$ implies $p^L < P(2K)$ and hence $D(p^L) > 2K$. Taking into consideration both possibilities, we write firm M's expected profit at $p = p^L$ as $p^L \min\{2K, D(p^L)\}$. Equation (6) is obtained from the condition $p^L \min\{2K, D(p^L)\} = p^H[D(p^H) - (n-2)K]$. ■

Proof that $P(nK) < p^L < p^H < P((n-2)K)$: Since Firm M would face a residual demand of 0 if it charges a price above $P((n-2)K)$, such a high price cannot be a solution to (5). Hence $p^H < P((n-2)K)$. It follows from $p^L \min\{2K, D(p^L)\} = p^H[D(p^H) - (n-2)K]$ that $p^L < p^H$. To prove that $p^L > P(nK)$, note that $K > \widetilde{K}_M/2$ implies $p^H[D(p^H) - (n-2)K] > 2KP(nK)$. Then $2Kp^L \geq p^L \min\{2K, D(p^L)\} > 2KP(nK)$. Thus $p^L > P(nK)$. ■

Proof of Proposition 2: From Lemmas 1, 3 and 4, we know that for K in this range, the merger turns a pure strategy equilibrium into a mixed strategy equilibrium. The post-merger prices are strictly higher because $p^L > P(nK)$. ■

Proof of Proposition 3: From Lemmas 1 and 3, we know that for K in this range, the merger turns a pure strategy equilibrium into a mixed strategy equilibrium. From $p^L \min\{2K, D(p^L)\} = p^H[D(p^H) - (n-2)K] > 0$ we know that $p^L > 0$. ■

Proof of Proposition 4(i): To show that $p^H > p^h$, note that p^H and p^h are solutions to the problem $\max_p p \cdot (D(p) - \kappa)$, with $\kappa = (n-2)K$ and $(n-1)K$. Conducting comparative statics on the first-order condition of this optimization problem, we find:

$$\frac{dp}{d\kappa} = \frac{1}{2D'(p) + pD''(p)} < 0. \qquad (A3)$$

Since $(n-2)K < (n-1)K$, (A3) implies that $p^H > p^h$.

We use (3) and (6) to prove that $p^L > p^l$. In the proof of Proposition 5 below, we show that $p^H[D(p^H) - (n-2)K] > 2p^h[D(p^h) - (n-1)K]$. Then using (3) and (6) we find:

$$p^L = \frac{p^H[D(p^H) - (n-2)K]}{\min\{2K, D(p^L)\}} \geq \frac{p^H[D(p^H) - (n-2)K]}{2K} > \frac{2p^h[D(p^h) - (n-1)K]}{2K} = p^l. \quad (A4)$$

■

Proof of Proposition 4(ii): We use F to denote a firm's mixed strategy before the merger, and F_M and F_O to denote the mixed strategy of the merged entity and an outsider, respectively. When characterizing the equilibrium mixed strategies, we use the fact that a firm's expected profit is constant and maximized at each price level p when the rivals play the equilibrium strategies. This implies $\pi_i(p_i, F^{n-1}) = F^{n-1}p(D(p) - (n-1)K) + (1 - F^{n-1})pK = p^l K$ in the pre-merger equilibrium. From this equation we obtain the pre-merger equilibrium strategy of each firm presented in (4).

For the post-merger equilibrium strategies F_M and F_O, we are able to derive explicit solutions in only a subset of cases. Fortunately, we do not need the complete set of solutions in order to prove our result. Since we have established that $p^H > p^h$ and $p^L > p^l$, we can conclude that F_M and F_O stochastically dominate F for any price below p^L or above p^h. What remains to be proven is that this result holds for any price in the range $(p^L,\ p^h)$.

We will proceed as follows. We will first characterize the post-merger equilibrium mixed strategies. Then we will compare the equilibrium strategies before and after the merger. Depending on the magnitude of $P((n-1)K)$ relative to p^H and p^L, there are three cases we must consider.

Case 1: $P((n-1)K) > p^H > p^L$.

The expected profits of firm M and each outsider can be written as:

$$\pi_M(F_M, F_O) = \int_{p^L}^{p^H} [F_O^{n-2}p(D(p) - (n-2)K) + 2(1 - F_O^{n-2})pK]dF_M = 2p^L K; \quad (A5)$$

$$\pi_O(F_M, F_O) = \int_{p^L}^{p^H} [F_M F_O^{n-3} p(D(p) - (n-1)K) + (1 - F_M F_O^{n-3})pK] dF_O = p^L K. \quad (A6)$$

We solve (A5) and (A6) to obtain:

$$F_M = 2^{-(n-3)/(n-2)} \left[\frac{(p - p^L)K}{p(nK - D(p))}\right]^{1/(n-2)} \quad , \quad F_O = \left[\frac{2(p - p^L)K}{p(nK - D(p))}\right]^{1/(n-2)} . \qquad (A7)$$

Case 2: $p^H > P((n-1)K) > p^L$.

In this case, the expected profits have to be expressed differently depending on whether the number of firms exceeds three. If $n = 3$, the expected profits of firm M and each outsider can be written as:

$$\pi_M(F_M, F_O) = \int_{p^L}^{P(2K)} [F_O p(D(p) - K) + 2(1 - F_O)pK] dF_M + \int_{P(2K)}^{p^H} [F_O p(D(p) - K) +$$

$$(1 - F_O)pD(p)] dF_M = 2p^L K; \qquad (A8)$$

$$\pi_O(F_M, F_O) = \int_{p^L}^{P(2K)} [F_M p(D(p) - 2K) + (1 - F_M)pK] dF_O + \int_{P(2K)}^{p^H} (1 - F_M)pK dF_O = p^L K.$$

(A9)

Using the above equations, we obtain

$$F_M = \frac{(p - p^L)K}{p(3K - D(p))}, \quad F_O = \frac{2(p - p^L)K}{p(3K - D(p))} . \qquad (A10)$$

for prices in the range $p^L < p < P(2K)$. (We omit the probability distributions for prices in the range $P(2K) < p < p^H$ as they are not needed for our proof). Note that (A10) is identical to (A7) for $n = 3$.

If $n \geq 4$, the expected profit of firm M can be written as

$\pi_M(F_M, F_O) = 2p^L K$

$= \int_{p^L}^{P((n-1)K)} [F_O^{n-2} p(D(p) - (n-2)K) + 2(1 - F_O^{n-2})pK] dF_M$

$+ \int_{P((n-1)K)}^{p^H} [F_O^{n-2} p(D(p) - (n-2)K) + (n-2)F_O^{n-3}(1 - F_O)p(D(p) - (n-3)K) +$

$2\big(1 - F_O^{n-2} - (n-2)F_O^{n-3}(1 - F_O)\big)pK] dF_M,$ (A11)

where F_O^{n-2} is the probability that firm M offers the highest price, and $(n-2)F_O^{n-3}(1-F_O)$ is the probability that it offers the second highest price. The expected profit of an outsider can be expressed as:

$\pi_O(F_M, F_O) = p^L K = \int_{p^L}^{P\left((n-1)K\right)} [F_M F_O^{n-3} p(D(p) - (n-1)K) + (1 - F_M F_O^{n-3})pK] dF_O$

$+ \int_{P((n-1)K)}^{p^H} [F_M F_O^{n-3} p \times 0 + (n-3)F_M F_O^{n-4}(1 - F_O)p(D(p) - (n-2)K)$

$+(1 - F_M F_O^{n-3} - (n-3)F_M F_O^{n-4}(1 - F_O))pK] dF_O,$ (A12)

where $F_M F_O^{n-3}$ is the probability that this outsider offers the highest price, and $(n-3)F_M F_O^{n-4}(1-F_O)$ is the probability that this firm offers the second highest price and firm M offers an even lower price. Solving (A11) and (A12), we find that F_M and F_O are the same as (A7) for prices in the range $p^L < p < P((n-1)K)$.

Case 3: $p^H > p^L > P((n-1)K)$.

In this case, we do not need an explicit solution for F_M and F_O because $P\big((n-1)\big)K > p^h$ implies that $p^L > p^h$, from which we know that F_M and F_O strictly dominate F stochastically at first order.

Therefore, we need to consider only Cases 1 and 2 to complete the proof. Recall that the relevant range of prices is $p^L < p < p^h$ and that $p^h < P((n-1)K)$. Thus, the relevant post-merger equilibrium strategies are given by (A7). We want to show that $F > F_M$ and $F > F_O$ for prices in the range $p^L < p < p^h$.

Next, we proceed to prove that $F > F_O$. Here we use the fact that the exponential function $f(x) = a^x$ is decreasing in x for $a \in (0, 1)$. Note that $1/(n-2) > 1/(n-1)$ and that in (A7),

$$0 < \frac{2(p - p^L)K}{p(nK - D(p))} < 1 \qquad (A13)$$

for $p \in (p^L,\ p^h)$. Thus, we have

$$F_O = \left[\frac{2(p - p^L)K}{p(nK - D(p))}\right]^{1/(n-2)} < \left[\frac{2(p - p^L)K}{p(nK - D(p))}\right]^{1/(n-1)}. \qquad (A14)$$

To complete the proof, we need to prove that $p - p^l > 2(p - p^L)$. This can be done by noting that $F(p^h) = 1 > F_O(p^h)$ implies

$$\frac{(p^h - p^l)K}{p(nK - D(p))} = 1 > \frac{2(p^h - p^L)K}{p(nK - D(p))}. \qquad (A15)$$

From (A15) we obtain $p^h - p^l > 2(p^h - p^L)$. Since the difference $(p - p^l) - 2(p - p^L)$ decreases in p, we have $p - p^l > 2(p - p^L)$ for any $p \in (p^L, p^h)$. The latter implies

$$\left[\frac{2(p - p^L)K}{p(nK - D(p))}\right]^{1/(n-1)} < \left[\frac{(p - p^l)K}{p(nK - D(p))}\right]^{1/(n-1)} = F. \qquad (A16)$$

From (A14) and (A16) we conclude that $F > F_O$ for $p \in (p^L, p^h)$.

Finally, it can be seen from (A7) that $F_O > F_M$ for all $p \in (p^L, p^h)$. Therefore, we conclude that $F > F_M$ for prices in the same range. ■

Proof of Proposition 5: We will prove that the merger is profitable for both firm M and each outsider in all three cases considered in Propositions 2, 3 and 4. In the case where $\widetilde{K}_M/2 < K \leq \widetilde{K}$, each firm earns a profit of $P(nK)K$ before the merger. After the merger, the expected profit of an outsider is equal to $p^L K$, which is higher than its pre-merger profit because $p^L > P(nK)$. The two insiders, on the other hand, earn an expected profit of $p^L \min\{2K, D(p^L)\}$ after the merger. It is clear that the merger is profitable for the insiders if $2K \leq D(p^L)$. In the case where $2K > D(p^L)$, we have $p^L > P(2K)$. Recall that the expected profit of firm M is also equal to $p^H[D(p^H) - (n-2)K]$. Optimization by firm M regarding p^H yields the first-order condition $D(p^H) - (n-2)K + p^H D'(p^H) = 0$, which implies $D(p^H) + p^H D'(p^H) = (n-2)K > 0$. Noting that $D + pD'$ is the first-order derivative of pD and that pD is concave, we have $d[pD(\cdot)]/dp > 0$ for $p < p^H$. Then $p^L > P(2K)$, implies $p^L D(p^L) > P(2K)2K$, which, in turn, is greater than $P(nK)2K$. Thus, the merger is profitable for the insiders.

In the case where $D(0)/(n-1) \leq K < D(0)/(n-2)$, each firm earns zero profit before the merger. The merger is profitable for both the insiders and outsiders because $p^L > 0$ implies that each firm earns a positive profit after the merger.

In the case where $\widetilde{K} < K < D(0)/(n-1)$, the expected profit of an outsider rises from $p^l K$ to $p^L K$. Hence the merger is profitable for each outsider. The expected profit of the two insiders is equal to $2p^h[D(p^h) - (n-1)K]$ before the merger and $p^H[D(p^H) - (n-2)K]$ after the merger. Since p^H is the solution to $\max_p p[D(p^H) - (n-2)K]$, we have $p^H[D(p^H) - (n-2)K] > p^h[D(p^h) - (n-2)K]$. Note that $D(p^h) < nK$ implies that $D(p^h) - (n-2)K > 2[D(p^h) - (n-1)K]$. Hence $p^H[D(p^H) - (n-2)K] > 2p^h[D(p^h) - (n-1)K]$. ■