Education as a Positional Good: 
The Role of Vouchers*

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Abstract

People's satisfaction from some goods and services depends on their relative as distinct from their absolute position as consumers. Such items are called "positional goods", and a restriction of their supply in the situation of general income growth is conducive to expenditure escalation as in an arms race. If education is a positional good in this sense, arrangements are needed that will best prevent such an outcome. The introduction of education vouchers of a value equal to the average per capita public school expenditure, it is argued, will only hinder not help. This is because some recipients will be tempted to obtain more education with marginal additions to their vouchers from their own pockets. Vouchers are thus welfare reducing because they encourage rather than discourage "arms race" situations. Using a formal median voter model we show that concerns over possible escalation of expenditure will prompt a majority of voters to reject a universal voucher system. We examine, as an alternative, a selective voucher system that will remove the escalation problem. Under this system only low-income families will receive vouchers. We demonstrate that the median voter will favor such a selective voucher system provided that the voucher-induced increase in competition lowers costs and/or improves quality of education.
I. Introduction

It has been observed that while there are many goods that are highly desirable in their own right, the supplies are such that only the wealthiest people can have them, and this regardless of the rate of growth of GNP. Hirsch (1976), who was the first to describe these goods as "positional", explained that they involve people in becoming concerned primarily with their relative as distinct from their absolute position as consumers. Consumer satisfaction with health care services, for example, depends not just on the absolute quality of those services but also on their relative quality. Each person tends to compare his services and another's over a period of time. The restriction of their supply in a situation of general growth of income might trigger off an escalation of expenditures (like an arms race). Once somebody attempts to purchase more, somebody else will match, or more than match, the increased expenditure in a quest to maintain his/her relative consumption position.

Does the phenomenon just described have implications for the methods of financing the social services? Some have suggested that "collectivization" of the supply of positional goods is preferable because it alone can be organized as to forestall the uncontrolled skyrocketing of expenditures. Thus switching from private fee-for-service physicians to prepaid group practice, it is argued, can be expected to curtail explosive medical expenditures, bearing in mind evidence that shows that health outcomes do not improve significantly with increased expenditures.
A second example, and one on which we wish to concentrate, is that of education. Those who view education as a positional good include Hirsch (1976), Jonathan (1990) and Frank (1995). The latter starts his discussion by referring to the finding of Hanushek (1986) that marginal increases in expenditures result in no significant marginal increases in educational achievement. This being so, those arrangements will be looked for that will be the most resistant to unnecessary expenditure accelerations (Frank, 1995).

The institution most capable of doing this, it is next argued, is the current "free" public school system. The reason is that this system imposes penalties on individual families who wish to buy more than average education for themselves. The chief penalty is that of forcing such families to "pay twice" for their education if they leave the public system. In other words, in order to buy more education they will have to resort to expensive private schooling and simultaneously forgo the "free" public schooling. Correspondingly, proposals to allocate to each family an education voucher of a value equal to the average per capita expenditure in the current public system are deemed inferior because some voucher recipients will then attempt to obtain more education by adding to the voucher from their own pockets. With the appearance of such "add ons", large numbers of families will emulate the practice so that each and every family will be troubled by the prospects of deterioration of their relative positions. When this concern spreads to the vast majority of families there will be irresistible
political pressure to increase the per capita expenditure in the public system. Once such pressure is successful, the expenditure escalation will be aggravated still further, and so on.

We do not enter the debate here about the extent to which education is a positional good. Instead we accept the proposition for analytical purposes, or, in other words, we treat it as a maintained hypothesis. Our objective is to draw out the implications of the positional good hypothesis in terms of the effects of voucher systems\(^1\) and their political feasibility under a majority voting rule. Using a formal model we show that a universal voucher system under which every family with school-age children receives vouchers will indeed lead to the kind of "arms race" discussed in the literature. Such escalation of education expenditure, however, will involve only families with relatively high income. Low income families will not want to participate in the "arms race" and will be made worse off by a move to the universal voucher system. As a result, in a community where income is unevenly distributed, a majority of voters will reject such a motion.\(^2\)

\(^1\) We are, of course, not the first to analyze the probable effects of introducing vouchers starting from a given equilibrium in traditional public school provision. Epple and Romano 1996(b), for instance, produce such an attempt but they are primarily interested in peer-group effects in education and study the introduction of universal vouchers exclusively. In contrast we focus on education as a positional good and on the differential effects of different types of vouchers.

\(^2\) Indeed, in the United States there were instances where such a motion was defeated in referenda (for example, 1993 California referendum for vouchers. See Epple and Romano 1996b).
As a solution to the "arms race" problem, we consider a selective voucher system under which only the low income families are entitled to receive vouchers. We demonstrate that this system has the best of both worlds. First, it accomplishes what a voucher system is supposed to achieve, namely, it introduces competition into the education sector which leads to reductions in price and improvements in the quality of education. Second, because vouchers are awarded to only those who do not participate in the "arms race", the selective voucher system avoids the problem of escalating expenditure that will likely occur under a universal voucher system. Therefore, a majority of voters will prefer a selective (pro-poor) voucher system over a universal voucher system.

The selective voucher system is beginning to receive empirical testimony in the current literature. West (1996) finds recently established selective pro-poor voucher plans operating in Colombia, Chile, Guatemala, the U.S.A., Puerto Rico and the U.K. Intellectual support, meanwhile, appears also to be growing. Vouchers to enable low income families exclusively to gain increased access to private schools are advocated, for instance, by Oakland (1994), West (1994) and Becker (1995). Oakland maintains that a case can be made for some redistribution generally in the provision of social services, but suggests that it is better accomplished by extending the welfare system to provide the poor with vouchers for selective government services such as education than using lump-sum and matching grants based on tax base
characteristics. Becker's recommendation is based partly on fiscal considerations, "but mainly because the bottom quarter or so of the population are most in need of better education" (p. 11). In addition, Becker quotes studies demonstrating, not only the superior performance of private over public schools, but also the finding that "students from disadvantaged backgrounds tend to gain the most from attending private schools". This fact, he observes, is not surprising "in light of the more extensive choices available to middle class and rich students" (p. 12).

Despite such arguments for a selective voucher system, so far there has been no formal analysis on the subject. The present paper accordingly attempts a rigorous theoretical investigation, in the light of the positional good concept, and one that may throw light on the recent emergence of the selective voucher system worldwide.

The paper is organized as follows. Section II analyzes government provision of positional goods/services in terms of a democratic political model. It assumes (a) that education is a positional good, (b) that education is a single voting issue, and (c) that the preferences that are met are primarily those of the median voter. In this section we assume the supply (cost) side of education is fixed. Section III considers the effects of switching to vouchers, first under a system of universal vouchers and second under a selective system wherein only low income families qualify to be recipients. Section IV drops the assumption that the supply side of education is fixed and examines the cost implications of
vouchers challenging the public school via new and effective competition. Section V offers our main conclusions.

II. The Model

Consider a community in which individuals have different income levels. Income is distributed in the interval \([\bar{Y}, \bar{Y}]\) with a probability density function \(\phi(Y)\). The average income level in this community can be expressed as 

\[ Y_a = \int_{\bar{Y}}^{\bar{Y}} Y \phi(Y) dY. \]

Distribution of income is such that \(Y_a\) is greater than the median income level, denoted by \(Y_m\). People have identical, homothetic tastes. Each individual consumes two goods: education and a numeraire good \((x)\). The quantity of education can be measured by a single index, \(e\), that takes into consideration both the quantity and the quality of education. A key feature of this model is that education is assumed to be a positional good. We follow Frank's (1985) seminal work on positional goods and assume that the utility function of an individual takes the form 

\[ U = U(e, x, R(e)), \]

where \(R(e)\) is an index measuring the percentile ranking of \(e\) in the population. Let \(f(e)\) denote the probability density function of \(e\). Then 

\[ R(e) = \int_{e}^{\bar{Y}} f(z) dz. \]

It follows that \(R'(e) = f(e) \geq 0\). Each individual's ranking, \(R(e)\), depends on the amount of spending on education by this individual as well as the amount of spending by all other individuals in this community. Following Frank, we shall make the following Cournot-type assumption: when choosing his own spending on education, each individual takes the spending of other individuals as fixed.

The assumption of homothetic preferences implies that
U(e,x,R(e)) = U(V(e,x),R(e))  \hspace{1cm} (1)

where V(e,x) is concave and homogenous of degree one in (e,x). Furthermore, U_\text{v} > 0, U_\text{R} > 0, U_{\text{Vv}} < 0, U_{\text{RR}} < 0, \text{ and } U_{\text{VR}} > 0. \text{ We assume that } U(V(e,x),R(e)) \text{ exhibits diminishing marginal rate of substitution between } e \text{ and } x.

The price of the numeraire good x is normalized to 1. The price of a unit of e is denoted by p. We assume that p is the same for public and private schooling. In this section and next, we assume that p remains constant as the community moves to a voucher system.

The focus of this analysis is on various regimes under which education can be provided. Our starting point is the public school system currently adopted in most jurisdictions in North America. Under this system, public schools are financed through tax revenues and, therefore, are "free" to individuals. At the same time, an individual has the option of going to a private school but he/she has to pay the full price of the private education out of his/her after-tax income. This is sometimes described as "paying twice" for education. For ease of presentation, we shall call this current public school system the PP regime. We shall then analyze what would happen if the community moved from PP to each of two modified regimes that involve vouchers. The first is a universal voucher system (the UV regime) under which all families and school-

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\[^3\text{In reality there is evidence to the contrary (see Toma 1996 and West 1996), but this assumption serves to simplify our analysis.}\]
age children qualify for vouchers. The second is a selective voucher system (the SV regime) under which only low-income individuals qualify for vouchers. All others in the school-age population are entitled to the conventional public schooling. They can also choose private schooling if they are prepared to "pay twice". We focus on the two issues: first, what will happen if this education system is reformed through the introduction of vouchers? Second, will a voucher system defeat the PP regime under the majority voting rule?

We assume that public education is financed by a flat tax on income, and that the level of public expenditure on education is determined by the majority voting rule. It is well known that under the PP regime a majority voting equilibrium may not exist (Epple and Romano 1996c, 79). In this paper, however, the PP regime is the reference point to which the UV regime and SV regime are compared. Therefore, we assume that, as the status quo, education is provided under a PP regime that is the outcome of the majority voting. To make the problem interesting, we assume that not all people use public education at the status quo.

Epple and Romano (1996c) contains a comprehensive analysis on the equilibrium under the PP regime. Here we adapt part of their analysis to our model and establish a number of basic facts about the PP regime that are necessary for our later analysis of vouchers. Let $t$ be the tax rate on income and $g$ be the amount of public education available to each person attending. Then the utility of a person who uses public education can be written as
U(V(g,(1-t)Y), R(g)). Then

\[
M(g,t) = \frac{U_{V}(V,R)Ve(g,(1-t)Y) + U_{R}(V,R)R'(g)}{YU_{V}(V,R)V_{e}(g,(1-t)Y)}
\]  

(2)

represents the slope of this person's indifference curve in the \((g,t)\) space. Diminishing marginal rate of substitution implies that \(\partial M/\partial g < 0\) and \(\partial M/\partial t < 0\). Following Epple and Romano (1996c), we focus on the following two cases:\(^4\)

a) \(\partial M/\partial Y < 0\) (Slope Declining in Income, or SDI);

b) \(\partial M/\partial Y > 0\) (Slope Rising in Income, or SRI).

For a given level of public education \(g\) and given tax rate \(t\), some voters choose to go to the public schools while others private schools. It can be shown that this choice depends on the income level of each individual.

**Lemma 1.**\(^5\) Given the values of \(t\) and \(g\), there exists a critical \(Y_p(g,t)\) such that individuals whose \(Y < Y_p(g,t)\) use public education and individuals whose \(Y > Y_p(g,t)\) use private education. Furthermore, \(\partial Y_p/\partial g > 0\) and \(\partial Y_p/\partial t > 0\).

Therefore, given \(g\) and \(t\) the fraction of individuals in this community who use the public education system is \(\int Y_p(g,t)\phi(Y)dY\). (To simplify notation we will drop the \(Y\) under the integral sign

\(^4\) See Epple and Romano (1996c) for a discussion on the implications and empirical relevance of these two alternative assumptions.

\(^5\) The proofs of all the lemmas and propositions in this paper are presented in Appendix.
from now on.) Notice that $g$ and $t$ are related through the government budget constraint: $pg\int^\infty y_p(g, t)\phi(Y)dY = ty_a$. On the left-hand side of the budget constraint is the per capita expenditure on public education and on the right-hand side the per capita tax revenue. From the budget constraint we can solve $t$ as a function of $g$: $t = t(g)$. Then we can define a new function $y_p(g) = y_p(g, t(g))$ and express the per capita expenditure on public education as a function of $g$ only: $pg\int^\infty y_p(g)\phi(Y)dY$. Obviously, as $g$ increases, the per capita expenditure on public education will increase as well. Define the increase in the per capita expenditure associated with a marginal increase in $g$ as "the marginal cost of public education", $MC(g)$. Then

$$MC(g) = \frac{d}{dg}\left[pg\int^\infty y_p(g)\phi(Y)dY\right] = p\int^\infty y_p(g)\phi(Y)dY + pg\phi(y_p(g))y_p'(g)$$

We assume that $MC(g)$ is nondecreasing in $g$. This assumption implies that $MC(g) \leq p$. Intuitively, if $g$ were so high that everyone uses the public education (i.e., if $y_p(g) = \bar{Y}$), the marginal cost of public education would be equal to the price of education $p$. For a smaller $g$, not everyone uses the public education system, in which case the assumption of nondecreasing marginal cost implies that $MC(g) \leq p$.

Differentiation of the government budget constraint $pg\int^\infty y_p(g)\phi(Y)dY = ty_a$ yields: $dt/dg = MC(g)/y_a$. In other words, an increase in $g$ by one unit has to be accompanied by an increase in the tax rate by $MC(g)/y_a$. 
Consider the median voter who has chosen the current level of \( g^* \) under the PP regime. Let \( Y_c \) denote the income of this median voter. Then the identity of the median voter depends on the sign of \( \partial M/\partial Y \). If SDI is satisfied, an individual's most preferred \( g \) is a monotonic function of \( Y \), in which case the individual with the median income is the median voter. On the other hand, if SRI is true, both low income individuals and high income individuals prefer a small or zero \( g \) while individuals in the middle of income spectrum prefer a large \( g \), a situation that Epple and Romano (1996c) characterize as "ends against middle." In this case the median voter is someone with an income less than the median income.

**Lemma 2.** Assume that a majority voting equilibrium exists. Then \( Y_c = Y_m \) if SDI is satisfied, and \( Y_c < Y_m \) if SRI is satisfied.

In what follows, we develop a diagrammatic illustration of the equilibrium under the PP regime. The diagrams will also be used in Section III to illustrate the effects of vouchers.

Let \( g^* \) and \( t^* \) be the level of public education and tax rate prevailing under the PP regime. Then \( g^* \) and \( t^* \) are the most preferred choice of the median voter. In other words, \( g^* \) and \( t^* \) are the solution to the problem:

\[
\max_{g,t} U(V(g,x),R(g))
\]

subject to \( pg\int_{Y_p(g)} Y \phi(Y) dY = tY_a \) and \( x = (1-t)Y_c \)

Combining the two constraints, we obtain:
\[ x + \left(\frac{Y_c}{Y_a}\right)pg \int_{Y_p(g)}^{Y_c(g)} \phi(Y) dY = Y_c \]  

(6)

The first-order condition to this problem implies:

\[ \frac{U_V(V,R)V_e(g^*,x^*) + U_R(V,R)R'(g^*)}{U_VV_x(g^*,x^*)} = \frac{Y_c}{Y_a}MC(g^*) \]  

(7)

The left-hand side of (7) represents the marginal rate of substitution between \( e \) and \( x \) when \( e \) is equal to \( g^* \) and is funded through tax revenue. The right-hand side is the "price" of \( g \) to the median voter. To see this, recall that tax rate has to be increased by \( MC(g)/Y_a \) in order to finance a marginal increase in \( g \). To the median voter, this increase in tax rate translates into an increased tax burden by the amount of \( (Y_c/Y_a)MC(g) \). Notice that \( (Y_c/Y_a)MC(g) < p \) because \( Y_c < Y_a \) and \( MC(g) \leq p \).

The solution to this first-order condition is illustrated in Figure 1. \( I_c \) is the median voter's indifference curve that goes through the point B. The slope of this indifference curve at \( e=g^* \) is given by the left-hand side of (7).

If the median voter had chosen to use a private school, he would have to finance his consumption of both \( e \) and \( x \) out of his after-tax income. Thus he would face a budget constraint:

\[ x + pe = (1-t^*)Y_c. \]  

(8)

This is represented by line HJ in Figure 1. Since, by assumption, \( g^* \) is positive in equilibrium, HJ must be everywhere below the indifference curve \( I_c \). (Otherwise, the median voter would be better off by using privately provided education, in which case he
should have chosen \( g = 0 \). The second line in Figure 1, line DKBF, is parallel to line HJ and goes through point B. It represents the equation

\[
x + pe = (1-t^*)Y_c + pg^*.
\]

Equation (9) would be the budget constraint of the median voter if he were free to spend an income of \((1-t^*)Y_c + pg^*\). The vertical distance between line HJ and line DKBF is the per capita public expenditure on education \( pg^* \). Since the slope of line DKBF is \(-p\), the length of the horizontal line HB is exactly equal to the units of public education purchased: \( g^* \). Notice that at point B the slope of DKBF is steeper than that of \( I_c \).

Lemma 1 implies that \( Y_p(g^*) \) is the critical income level that divides individuals who use public education and individuals who use private education. It is clear that \( Y_p(g^*) > Y_c \). The choice of the individual with this critical income level is illustrated in Figure 2. If he uses the public education, he will consume \( g^* \) units of education for "free" and his utility level is given by the indifference curve \( I_{pp} \). On the other hand, if he uses private education, he will purchase \( e^* \) units out of his after-tax income \((1-t)Y_p(g^*)\). As shown in Figure 2, the indifference curve that goes through the point \((g^*, (1-t)Y_p(g^*))\), \( I_{pp} \), is tangent to the private education budget line \( x + pe = (1-t)Y_p(g^*) \), implying that he is indifferent between either consuming \( g^* \) units of public education or \( e^* \) units of private education. Diminishing marginal rate of substitution implies that the slope of \( I_{pp} \) at \( g^* \) is greater than \( p \), the slope of \( I_{pp} \) at \( e^* \).
For an individual with income level $Y' > Y_p(g^*)$, a portion of the private education budget line $x + pe = (1-t^*)Y'$ will lie above the indifference curve that goes the point $(g^*, (1-t^*)Y')$. To him, private education is the better alternative.

III. Vouchers

In this section we consider the effects of modifying the PP regime to a regime that allows education vouchers. We shall first consider a universal voucher system (the UV regime) under which every family with school-age children is entitled to vouchers. Then we shall propose a selective voucher system (the SV regime) under which only low income families receive vouchers. Under both voucher regimes those entitled to a voucher have the option to supplement it with additional purchase of education. We assume that a decision to switch regimes is made through a two-stage voting process. At stage 1, voters decide whether to switch from the PP regime to a voucher system (either UV or SV regime). If the outcome is "yes," they proceed to stage 2 and decide the values of $g$ and $t$ under the voucher system. In the following analysis of each voucher system, we shall begin by studying the possible effects on voters of different income levels should a switch to the voucher system be made. Once we know how different voters would fare under the voucher system, it is easy to predict the outcome of stage 1 voting.

III.1 UV Regime

The UV regime differs from the PP regime in at least two aspects.
First, the UV regime gives individuals more choices in the kinds of schools they want to attend. They can spend their vouchers in the private schools or public schools. This will increase competition among schools and will tend to improve the efficiency of schools. We shall explore the implications of this aspect in the next section. Second, the UV regime is universal in the sense that every family with school-age children is entitled to a voucher. Our analysis in this section will focus on this second aspect.

Before we present the results from our formal model, it is instructive to conduct a heuristic analysis on the UV regime using Figure 1 and Figure 2. In this diagrammatic approach, we consider a restricted version of the UV system where only those who were previously using the public education will be entitled to vouchers. In other words, those who were previously using private schools, or can be regarded as the "incumbent" private school population, are for now administratively precluded from the voucher option. (This assumption will later be dropped.) Under this restricted version of the UV regime the government budget constraint remains the same as under the PP regime, and at the tax rate $t^*$, the value of the voucher will be exactly equal to $pg^*$.

As will be shown in Proposition 1 the median voter under the UV regime is not necessarily the median voter under the PP regime. For simplicity, in this diagrammatic analysis we restrict our attention to the case where the median voters under these two regimes have the same income. In Figure 1 the value of the voucher, $pg^*$, can be represented by the length HD. Under the UV
regime, the median voter can use the voucher to obtain the first $g^*$ units and pay $p$ dollars for each additional unit beyond $g^*$. His budget constraint under this system is presented by HBF in Figure 1. Recall that BF is steeper than $I_c$ at point B. As a result, any point on the segment BF (not including B) will generate a lower utility level than at point B. The median voter will not buy any supplement even though he is allowed to do so. The assumption of identical and homothetic preferences then implies that individuals with lower income levels will not purchase any supplement, either.

Intuitively, these individuals do not want to purchase any supplements because their income levels are below the average income $Y_a$ (see Lemma 2). The public education they receive is subsidized by the individuals with above-average income. As a result, their marginal rate of substitution at $g^*$ is lower than the true price of education $p$. Under a voucher system, the median voter faces the true price of education at the margin. Hence he has no incentive to increase his consumption of education. In fact, he would want to reduce it if he could sell a portion of his voucher at price $p$ (see the BK line in Figure 1).

Let $Y_{uv}$ be the income level that divides those who buy supplements and those who do not. The preceding discussion implies that $Y_{uv} > Y_c$. The curve $I_{uv}$ in Figure 2 is an indifference curve of the individual with income $Y_{uv}$. It is tangent to the budget line $x + pe = (1-t^*)Y_{uv} + Pg^*$ at $e = g^*$. This individual has the highest income level among those who do not purchase any supplements. Individuals with a higher income want to consume more
than the g* units purchased with the voucher and hence purchase supplements. Recall that I_{pp} in Figure 2 is an indifference curve of the individual with income Y_p(g*) and that the slope of I_{pp} at g* is greater than p. The assumption of identical, homothetic preferences implies that I_{uv} is below I_{pp}, which, in turn, implies that Y_{uv} < Y_p(g*).

Therefore, individuals with income levels higher than Y_{uv} but lower than Y_p(g*) would increase their expenditure on education and hence switch from public education to private education if the UV system were introduced. The escalation of expenditure by these individuals would affect the behaviour of both low income and high income individuals. Notice that in the discussion so far we have implicitly assumed that the positional good index R remains constant. However, as those middle income individuals increase their expenditure beyond Pg*, the percentile ranking of those who continue to consume g*, measured by R(g*), is lower. This will make these individuals whose income is below Y_{uv} worse off. On the other hand, as more families switch to private schools, the percentile ranking of those who use private education under the PP regime, (individuals whose income is above Y_p(g*)), would drop unless they increase their expenditure on education as well. As a consequence, there would be an "arms race" among those with relatively high income levels (Y>Y_{uv}) as they increase their expenditure in an attempt to maintain their relative rankings.\(^6\)

\(^6\)It may be argued that privately volunteered supplements ("add ons") to vouchers could be legally prohibited, in which case the "arms race" fear would be resolved. Administratively, however,
These conclusions are confirmed by our analysis of the formal model. In the formal analysis we no longer assume that vouchers are restricted to previous users of public education. Instead, we consider a truly universal voucher system in which everyone is entitled to a voucher. This means that a given amount of education expenditure has to be shared equally among all individuals of the community. It also means that individuals who were using private education receive a windfall in the form of vouchers. As a result, the government budget constraint now changes to: \( pg = tY_a \), which implies \( t = pg/Y_a \). We shall use \( s \) to denote the supplement purchased by an individual. Then an individual's most preferred \( g \) is solved from

\[
\max_{g \geq 0, s \geq 0} U(V(g+s,Y-\frac{Ypg}{Y_a}-ps),R(g+s))
\]

From this optimization problem we can determine the relationship between an individual's most preferred \( g \) and his income level, and thus the identity of the median voter under the UV regime. Let \( Y_{cv} \) denote the income of the median voter under the UV regime. Recall that the income of the median voter under the PP regime is \( Y_c \).

**Proposition 1.**

Under the UV regime the median voter has the same income as the median voter under the PP regime (i.e., \( Y_{cv} = Y_c \)) if SDI holds, but his income is lower than that of the median voter under the PP regime (i.e., \( Y_{cv} < Y_c \)) if SRI holds.

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this would be extremely costly and difficult, especially since it is known that, at the margin, parents can, and do, substitute their own private donations for tuition fees. (Blewett, 1988.)
Therefore, under the UV regime the median voter has an income level equal to \( Y_m \) if SDI is true and an income level less than \( Y_m \) if SRI is true. Since \( Y_m \) is lower than the average income \( Y_a \), the median voter's income is less than \( Y_a \) as well. As a result, Proposition 2.

Under the UV system, individuals whose income levels are less than or equal to that of the median voter \((Y \leq Y_{cv})\) will not supplement their vouchers \((i.e., s = 0)\).

From the government budget constraint we can see that a marginal increase in \( g \) has to be financed by an increase in tax rate by \( p/Y_a \). To the median voter, this translates into an increase in tax by \( pY_{cv}/Y_a \). Recall that under the PP regime the "price" of \( g \) to this voter is \((Y_{cv}/Y_a)MC(g)\). Since \( MC(g^*) < p \), the voter faces a higher "price" of \( g \) under the UV regime than under the PP regime. This is not surprising considering that under the UV regime an increase in \( g \) will be applicable to all individuals rather than only those with income less than \( Y_p(g^*) \). The voter responds to this increase in the "price" of \( g \) in the following way.

Proposition 3.

Under the UV regime, the level of public education would be lower and the tax rate would be higher than under the PP regime, i.e., \( g_{uv} < g^* \) and \( t_{uv} > t^* \).

The next proposition pertains to the situation illustrated in
Figure 2.

**Proposition 4.**

There exists a critical income level $Y_{uv}$ such that individuals whose $Y > Y_{uv}$ supplement their voucher and individuals whose $Y < Y_{uv}$ do not. Furthermore, $Y_{cv} < Y_{uv} < Y_p(g^*)$.

Propositions 3 and 4 imply that low income individuals with $Y < Y_{uv}$ will lose from the switch from the PP regime to the UV regime. Under the UV regime, they will pay more tax yet receive less public education in return. Their relative position, measured by $R(g)$, will worsen as well because fewer people will use the public education. Therefore, if asked to choose between the PP regime and the UV regime, individuals with income less than or equal to $Y_{uv}$ will unambiguously vote against the switch from the PP regime to the UV regime.

The choice of those with higher income, however, is not as obvious. The high income individuals who use private education under the PP regime (whose $Y > Y_p(g^*)$) no longer have to "pay twice" for education and they receive a windfall in the form of vouchers. However, this windfall is dissipated, at least in part, by the "arms race" induced by the increased spending of middle-income individuals (those with an income between $Y_{uv}$ and $Y_p(g^*)$). As a result, it is not clear whether they will join the low income individual in voting against the switch to UV system. To individuals whose income falls between $Y_{uv}$ and $Y_p(g^*)$, the UV regime allows them to choose the exact amount of e that would maximize
their utility. This gain is, again, (at least partially) offset by the "arms race".

**Proposition 5.**
A motion to replace the PP regime by the UV regime will be rejected by a majority of voters if either a) SDI holds, or b) SRI holds and \( Y_{uv} \geq Y_m \).

The possibility of escalating expenditure caused by a switch to the UV system is discussed in Frank (1996). Using a diagram similar to Figure 1 he argues that a voucher system will lead to increased expenditure on education for many families, "possibly the vast majority of families" (page 182). Frank reaches this conclusion by assuming that a representative individual faces a situation represented by point K (rather than point B) in Figure 1. Our analysis reveals this assumption is not appropriate if we interpret Frank's representative individual as the median voter.\(^7\) The median voter has no desire to increase his consumption of \( e \) after the UV regime is adopted. In fact in equilibrium he will reduce his consumption of education. Despite this observation, Frank is still correct in his contention that, given his assumptions, a move to the UV regime will trigger an "arms race" in education expenditure.

Here we push our analysis further than that of Frank. We show that the concern over expected escalating expenditure will prompt

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\(^7\) A representative agent is the agent whose choices are the same as the ones chosen by the community as a whole. In our model the median voter plays precisely this role.
the median voter to reject the move from the PP regime to the UV regime.

III. 2 SV Regime

Consider now a selective voucher system where only low income individuals are entitled to receive vouchers. Here low income is defined as the level below \( Y_c \), the income level of the median voter under the PP regime. From the above analysis, the effects of this SV regime are easy to ascertain. Because \( Y_c < Y_p(g^*) \), users of private education do not receive vouchers. As a result, the government budget constraint is not affected by a move from the PP regime to such a SV regime. Furthermore, by restricting vouchers to low income individuals, individuals who would have engaged in an "arms race" do not receive vouchers under the SV regime. Hence a move from the PP regime to the SV regime will not lead to any change in expenditure or tax rate. This implies that \( R(g^*) \), the ranking of those with \( Y \leq Y_c \), is not affected by this change. They will continue to consume the \( g^* \) units purchased with the voucher.

**Proposition 6.**

Given the price \( p \), the SV regime is tied with the PP regime under the majority voting rule.

Notice that given the assumption of constant price \( p \), everyone is as well off under the SV regime as under the PP regime. In other words, there is no gain from moving from the PP regime to the SV regime. This, of course, will not be true if the increased competition brought about by the voucher system brings down the
price of education (or improves quality). We discuss this issue in next section.

IV. The Production Side

In the above analysis, we have assumed that the supply side of the education system is fixed. Hence the price of education, \( p \), is assumed to be constant before and after the introduction of voucher system. In this section we discuss the effects of a voucher system on the supply side. We view schools as firms that produce output (education) using a production technology subject to decreasing returns to scale. We assume that the public school and the private school have the same cost structure.

Under PP regime, the public schools as a whole have a monopoly power over the segment of the market in which individuals' demand for education is less than or equal to \( g^* \) units (see Figure 1). This is because public education is "free" to everyone in the community. Private schooling, on the other hand, has to be paid through after-tax income. As a result, private schools can only supply the residual market in which buyers want more than \( g^* \) units of education. Since the public schools have a market share of more than 50 percent, it is reasonable to assume that they are the price-setters in the market. The private schools, on the other hand, are price followers in the sense that they take the price as given when deciding their output. We view this situation as similar to the Stackelberg leadership model where the public schools form a cartel and act as a price leader while the private
schools form a competitive fringe. It is clear that in this situation the equilibrium price will be above marginal cost.

One consequence of the voucher system is the introduction of competition into the market segment previously monopolized by public schools. Now private schools can compete with public schools to supply $g^*$ units paid for by the vouchers. The increased competition will bring down the price of education $p$.\(^8\) This will bring benefits to all consumers of education. It can be shown that in equilibrium, the amount of education provided by the public funds will increase ($g > g^*$) and the tax rate will decrease ($t < t^*$). Clearly, the individuals who use public education will benefit from such a change. The users of private education also benefit from this change due to lower price and lower tax rate.

A move from the PP regime to the UV regime, will bring down $p$. But to the median voter this effect is partially or even completely offset by the negative effects of an "arms race" among the high income individuals. A move to the SV regime, on the other hand, does not have this second negative effect. Therefore our next proposition logically follows:

**Proposition 7.**

\(^8\) Recall that $e$ is an index that takes into consideration of both quality and quantity of education. A fall in the price $p$ can be interpreted both as a fall in the cost of education and as an increase in the quality of education. In reality, It is more likely that a fall in $p$ will manifest itself in the form of improvement in quality. New empirical evidence has been published in the 1990s showing that the introduction of competition via increased use of private schooling leads to improved public school performance. (See Couch et al., 1993; Minter-Hoxby, 1994; and Borland and Hoxsen, 1996.)
Assuming that a voucher system can be expected to reduce the price of education, the SV regime will defeat the PP regime, and it will defeat the UV regime if a) SDI is true, or if b) SRI is true and $Y_{uv} > Y_m$.

The SV regime has the best of both worlds. It introduces competition into the education sector, and hence reduces the price and improves the quality of education. At the same time, it avoids the positional good problem of escalating expenditure that will likely occur under the UV regime.\(^9\)

V. Conclusion

Our model predicts that, assuming education is a positional good, the median voter will reject a proposed move from the current system of public plus private schools (PP) to a regime of vouchers that are universally available to families with school-age children (UV). The median voter will be indifferent to any proposal for a selective voucher (SV) available only to low income families, but only if we assume that production costs can be expected to remain fixed. Because, however, the selective voucher will allow private

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\(^9\) Since the introduction of the SV regime will be an efficient move that reduces the price and improves the quality of education, the question arises why the median voter approved of the inefficient alternative in the first place. One answer may be offered in terms of initially imperfect voter information. The public monopoly in education took time to develop, as did the growth of the education bureaucracy and its constant pressure for consolidation.
schools to compete for low income students traditionally in public schools, the public school monopoly will be weakened and costs will come down all round (and/or quality will improve all round). The median voter in this situation will be in favor of the adoption of SV. He will be opposed to a universal voucher, however, because, despite the fact that the new competition will reduce price, this effect will be counterbalanced by the negative effects of an "arms race" among high income families. The SV, in other words, avoids the positional good problem of escalating expenditures that do little or nothing to increase output, whereas the UV regime fails to avoid it. The SV regime could result in higher expenditures but this time as a welfare gain in the form of greater output via increased competition.
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Minter-Hoxby, C. (1994) "Do Private Schools Provide Competition for Public Schools?" Unpublished manuscript, Department of Economics, MIT
Appendix

Proof of Lemma 1.

Given the values of \( g \) and \( t \), an individual with income \( Y \) can either use public education and consume the bundle \((g, (1-t)Y)\), or use private education and consume the utility-maximizing bundle associated with the disposable income \((1-t)Y\). Hence his choice of education system is determined by comparing \( U(V(g, (1-t)Y), R(g)) \) with \( \max_e U(V(e, (1-t)Y-pe), R(e)) \). Using the envelope theorem we can show that the latter increases with \( Y \). Let \( Y_p(g,t) \) be the income of the individual who is indifferent between the public and private education, that is, \( Y_p(g,t) \) satisfies

\[
U(V(g,(1-t)Y_p), R(g)) = \max_e U(V(e,(1-t)Y_p-pe), R(e)). \tag{A1}
\]

Then individuals with \( Y < Y_p(g,t) \) prefers public education to the private education and individuals with \( Y > Y_p(g,t) \) prefers private education to the public education.

To prove that \( \frac{\partial Y_p}{\partial g} > 0 \) and \( \frac{\partial Y_p}{\partial t} > 0 \), consider the individual with income \( Y_p(g,t) \). Let \( e^* \) be the solution to the maximization problem on the right-hand side of (A1). It is obvious from equation (A1) that \( e^* > g \). We will use \( U(e^*) \) and \( V(e^*) \) to denote the values of \( U(\cdot) \) and \( V(\cdot) \) when he chooses private education and \( U \) and \( V \) to denote the values of the same functions when he chooses public education. Since \( Y_p(1-t) - pe^* < Y_p(1-t) \) and \( e^* > g \), the marginal utility of \( x \) is higher if he chooses private education than if he chooses public education, i.e., \( U_v(e^*) V_x(e^*) > U_v V_x \). Comparative statics on equation (A1) reveals that
and \( \partial Y_p / \partial t = Y_p / (1 - t) > 0 \).

**Proof of Lemma 2.**

To determine the identity of the median voter, we must find out how each individual's utility-maximizing value of \( g \) is related to his income. Obviously, an individual's most preferred \( g \) is determined by comparing his maximum utility from using the public education

\[
\max_g U(V(g, Y - \frac{Y}{Y_a}) pg \int_{Y_p(g)}^{Y}(Y) \phi(Y) dY), R(g)) \tag{A3}
\]

with the utility from using the private education

\[
\max_{g,e} U(V(e, Y - \frac{Y}{Y_a}) pg \int_{Y_p(g)}^{Y}(Y) \phi(Y) dY - pe), R(e)) \tag{A4}
\]

First, we establish that high income individuals prefer \( g=0 \) to \( g>0 \). Consider the individual who has the highest income in this community, \( \bar{Y} \). From Lemma 1 it is clear that if he should choose the public education, his most preferred \( g \) would be large enough that everyone would use public education, i.e., \( \int_{Y_p(g)}^{Y}(Y) \phi(Y) dY = 1 \). Then \( p(\bar{Y}/Y_a) \int_{Y_p(g)}^{Y}(Y) \phi(Y) dY > p \); in other words, public education is more costly than private education to this individual. Meanwhile, his top income level ensures that \( R(e) = 1 \) should he choose private education \( (g=0 \text{ and } e>0) \). Therefore, private education and hence \( g=0 \) is the most preferred choice of this individual. By continuity, there exists an income level \( \hat{Y} \) \( (< \bar{Y}) \) such that the most preferred \( g \) is equal to 0 for all individuals with \( Y > \hat{Y} \).
Second, we demonstrate that $\hat{Y} > Y_a$. We achieve this by showing that those with $Y \leq Y_a$ prefers $g > 0$ to $g = 0$. Consider again the optimization problems (A3) and (A4). For an individual with $Y < Y_a$, $(Y/Y_a) \int Y \phi(Y) dY < p$ and $R(g) \geq R(e)$ (because $Y_p(g) > Y$). The value of (A3) is always greater than the value of (A4). The same is true for an individual with $Y = Y_a$ because $p \int Y \phi(Y) dY \leq p$, $R(g) \geq R(e)$, and at least one of these two holds with strict inequality. Therefore, the most preferred $g$'s of all individuals with $Y \geq Y_a$ must satisfy $g > 0$.

Third, we show that, among those who prefer $g > 0$ to $g = 0$, an individual's most preferred $g$ decreases with his income if SDI holds but increases with his income if SRI holds. The utility of these individuals is represented by (A3). The first order condition to this maximization problem is $Y_a M(g, (pg/Y_a) \int Y \phi(Y) dY) - MC(g) = 0$, where $M(g, t)$ is defined in equation (2) and $MC(g)$ in equation (3). Comparative statics on this condition reveals that

$$\frac{\partial g}{\partial Y} = \frac{Y_a (\partial M/\partial Y)}{MC'(g) - Y_a (\partial M/\partial g)}$$  \hspace{1cm} (A5)$$

The denominator is positive because $\partial M/\partial g < 0$, $\partial M/\partial t < 0$ and $MC'(g) > 0$. The sign of $\partial g/\partial Y$ is negative if $\partial M/\partial Y < 0$ (SDI) and positive if $\partial M/\partial Y > 0$ (SRI).

Therefore, if SDI holds, an individual's most preferred $g$ decreases with income, with $g = 0$ for the individuals with very high income. In this case, the median voter is the individual with the median income, i.e., $Y_c = Y_m$. On the other hand, if SRI holds, an individual's most preferred $g$ increases with income for those who
prefer \( g > 0 \) to \( g = 0 \). In this case, a majority voting equilibrium may not exists because individuals whose income is between \( Y_a \) and \( \hat{Y} \) may have two local maxima to their utility maximization problem, \( g > 0 \) in one and \( g = 0 \) in the other. To rule out this possibility and ensure the existence of an equilibrium we have to assume that individuals with \( Y > Y_a \) has a unique most preferred \( g \). Then we can re-interpet \( \hat{Y} \) as the income level that divides those who prefer \( g > 0 \) and those who prefer \( g = 0 \). The median voter, thus, has an income level \( Y_c \) defined by \( \int_{Y}^{Y_c} \phi(Y) dY + \int_{Y}^{\hat{Y}} \phi(Y) dY = 1/2 \). Clearly, \( Y_c < Y_m \).

**Proof of Proposition 1.**

Again we consider each individual's most preferred \( g \). The first-order conditions to the maximization problem (10) presented in the text are

\[
\frac{\partial U}{\partial g} = U_v V_e + U_R R'/U_v V_x P(Y/Y_a) \leq 0; \quad g \geq 0; \quad \frac{\partial U}{\partial g} g = 0 \quad (A6)
\]

\[
\frac{\partial U}{\partial s} = U_v V_e + U_R R'/U_v V_x P \leq 0; \quad s \geq 0; \quad \frac{\partial U}{\partial s} s = 0 \quad (A7)
\]

For an individual with \( Y > Y_a \), \( p(Y/Y_a) > p \). The above two conditions imply that \( g = 0 \) and \( s > 0 \). Conversely, for an individual with \( Y < Y_a \), (A6) and (A7) imply that \( g > 0 \) and \( s = 0 \).

Consider those whose \( Y < Y_a \) and hence whose optimal \( g > 0 \). Condition (A6) implies that \( Y_a M(g, pg/Y_a) - p = 0 \). Comparative statics on this condition reveals that

\[
\frac{\partial g}{\partial Y} = -\frac{\partial M/\partial Y}{\partial M/\partial g} \quad (A8)
\]
The denominator is negative. Then the sign of $\partial g/\partial Y$ is negative if $\partial M/\partial Y < 0$ (SDI) and positive if $\partial M/\partial Y > 0$ (SRI).

Therefore, if SDI holds, an individual's most preferred $g$ decreases with his income, with $g=0$ for the individuals with very high income. The median voter has an income $Y_{cv} = Y_m$, which is the same as under the PP regime.

On the other hand, if SRI holds, an individual's most preferred $g$ increases with his income for those who prefer $g>0$ to $g=0$. In this case, individuals at the two ends of the income distribution want a small or zero $g$ but individuals in the middle prefer a large $g$. The median voter has an income level defined by $\int_{Y}^{Y_{cv}} \phi(Y) dY + \int_{Y_a}^{Y} \phi(Y) dY = 1/2$. Recall from the proof of Lemma 2 that $Y_c$ is defined by $\int_{Y}^{Y_{c}} \phi(Y) dY + \int_{Y}^{Y} \phi(Y) dY = 1/2$ and that $Y > Y_a$. It follows that $Y_{cv} < Y_c < Y_m$.

**Proof of Proposition 2.**

Proposition 1 implies that $Y_{cv} < Y_a$. Conditions (A6) and (A7) implies that the median voter will choose $s = 0$. Thus, $\partial U/\partial s \leq 0$ when $Y = Y_c$. It can be shown, by differentiating (A7), that $\partial^2 U/\partial Y \partial s > 0$. Therefore, $\partial U/\partial s < 0$ for individuals whose $Y < Y_c$. They will also choose $s = 0$.

**Proof of Proposition 3.**

Since $\partial M/\partial t < 0$ and $\int^{Y_{p}(g)} \phi(Y) dY \leq 1$, $M(g, (pg/Y_a)\int^{Y_{p}(g)} \phi(Y) dY) > M(g, pg/Y_a)$ for the same income $Y$. If SRI is true, $Y_c > Y_{cv}$, in which case $M(g, (pg/Y_a)\int^{Y_{p}(g)} \phi(Y) dY)$ evaluated at $Y_c$ is still greater than $M(g, pg/Y_a)$ evaluated at $Y_{cv}$. On the other hand, recall from the proof of Lemma 2 that $Y_a M(g, (pg/Y_a)\int^{Y_{p}(g)} \phi(Y) dY) = \ldots$
MC(g) at $g = g^*$, and from the Proof of Proposition 1 that $M(g, pg/Y_a) = p$ at $g = g_{uv}$. Since $MC(g) \leq p$ and $\partial M/\partial g < 0$, to satisfy these two equations it is necessary that $g^* > g_{uv}$.

To prove that $t^* < t_{uv}$, let $g'$ be the solution to the equation $Y_aM(g, (pg/Y_a)\int Y\phi(Y) dY) = p$. Since $M(g, (pg/Y_a)\int Y\phi(Y) dY) > M(g, pg/Y_a)$, $\partial M/\partial g < 0$ and $\partial M/\partial t < 0$, we have $g' > g_{uv}$. Then $Y_aM(g', t^*) = Y_aM(g_{uv}, t_{uv}) = p$ implies that $t^* < t_{uv}$.

Proof of Proposition 4.

First, we prove that given the existence of $Y_{uv} \in (Y, \bar{Y})$, all individuals with $Y < Y_{uv}$ chooses $s = 0$ and all individuals with $Y > Y_{uv}$ chooses $s > 0$. Given $g_{uv}$ and $t_{uv}$, an individual's utility maximization problem is:

$$\max_{s \geq 0} U(V(g_{uv} + s, (1-t_{uv})Y-ps, R(g_{uv} + s))$$ (A9)

The first-order conditions are that $\partial U/\partial s \geq 0$, $s \geq 0$ and $s(\partial U/\partial s) = 0$. By the definition of $Y_{uv}$, $\partial U/\partial s = 0$ at $s = 0$ and $Y = Y_{uv}$. It can be verified that $(\partial^2 U/\partial Y \partial s) > 0$. Thus, for individuals whose $Y < Y_{uv}$, $\partial U/\partial s < 0$ at $s = 0$, in which case their utility maximization choice is $s = 0$. Conversely, for individuals whose $Y > Y_{uv}$, $\partial U/\partial s > 0$ at $s = 0$, in which case their utility maximization choice is $s > 0$.

Second, we establish that under the UV regime the individual with $Y = Y_p(g^*)$ will choose $s > 0$. To simplify notations in this proof we will use $Y_p$ as a shorthand notation for $Y_p(g^*)$. Recall that $Y_p$ is the income level of the individual who, in the PP equilibrium, is indifferent between public education and private education. Hence $Y_p$ satisfies:
\[ U(V(g^*, (1-t^*)Y_p), R(g^*)) = \max_e U(V(e, (1-t^*)Y_p - pe), R(e)) \]  \hspace{1cm} (A10)

Let \( e_p \) be the solution to the maximization problem on the right-hand side of (A10). A necessary condition for (A10) to hold is that \( e_p > g^* \).

Under the UV regime, the individual has an effective after-tax income of \((1-t_{uv})Y_p + pg_{uv}\), which he can spend on education \( g_{uv} + s \) \((s \geq 0)\) and the numeraire good \( x \). Substituting \( t_{uv} = pg_{uv}/Y_a \) for \( t_{uv} \), we can re-write it as \( Y_p - pg_{uv}(Y_p - Y_a)/Y_a \). On the other hand, the individual's after-tax income under the PP regime is \((1-t^*)Y_p\), which after substituting the government budget constraint for \( t^* \), can be written as \( Y_p - pg^*(Y_p/Y_a)\int Y_{p}\phi(Y)dY \). Since \( Y_p\int Y_{p}\phi(Y)dY < \int Y_{p}\phi(Y)dY < Y_a \), we have \( Y_p - Y_a < Y_p\int Y_{p}\phi(Y)dY \). This, along with that \( g_{uv} < g^* \), implies that \((1-t_{uv})Y_p(g^*) + pg_{uv} > (1-t^*)Y_p(g^*)\). Thus, the individual has a higher effective after-tax income under UV regime than under the PP regime. This implies that his consumption of education under the UV regime, \( g_{uv} + s \) exceeds \( g^* \). Since \( g^* > g_{uv} \), we have \( s > 0 \).

Thus, we can conclude that \( Y_{uv} \) should be less than \( Y_p \). On the other hand, we know that \( s=0 \) for the median voter whose income is \( Y_{cv} \). It follows that \( Y_{uv} \) falls between \( Y_{cv} \) and \( Y_p(g^*) \).

**Proof of Proposition 5.**

If SDI holds, \( Y_{cv} = Y_m \), which by proposition 2 implies \( Y_m < Y_{uv} \). Thus, under both condition a) and condition b) of the proposition, \( Y_m < Y_{uv} \). In both cases, the bottom 50\% of the population does not supplement their vouchers. Furthermore, under the UV regime g is
lower and t is higher than under the PP regime. They are worse off under the UV regime than under the PP regime. They will defeat the motion to replace PP regime with UV regime.

**Proof of Proposition 6.**

Given that $Y_c < Y_p(g)$, the government budget constraint under the SV regime is the same as under the PP regime: $tY_a = pg\int^{Y_p(g)} \phi(Y) dY$. Consider those individuals who receive vouchers. Their most preferred $g$ and $s$ are solved from:

$$\begin{align*}
\max_{g \geq 0, s \geq 0} & \quad U(V(g+s, Y - \frac{Y}{Y_a} pg\int^{Y_p(g)} \phi(Y) dY - ps), R(g+s)) \\
\text{subject to} & \quad g > 0; s > 0; \frac{\partial U}{\partial g} g = 0 \quad (A12)
\end{align*}$$

The first order conditions to this problem are:

$$\begin{align*}
\frac{\partial U}{\partial g} &= U_v V_e + U_R R' - U_v V_x (Y/Y_a) MC(g) \leq 0; g \geq 0; \frac{\partial U}{\partial g} g = 0 \\
\frac{\partial U}{\partial s} &= U_v V_e + U_R R' - U_v V_x p \leq 0; s \geq 0; \frac{\partial U}{\partial s} s = 0 \\
\end{align*}$$

Since $Y_c < Y_a$ and $MC(g) \leq p$, the most preferred choices of those who receive vouchers are such that $g > 0$ and $s = 0$. Given that no one will supplement the voucher he receives, there is no change in each individual's most preferred $g$ if the PP regime is replaced by the SV regime. The median voter under the SV regime has the same income level as the median voter under the PP regime (i.e., $Y_c$). He will choose the same level of public expenditure and tax rate, $g^*$ and $t^*$. Hence, there is no change in both public expenditure and private expenditure on education if the PP regime is replaced by the SV regime. Everyone is indifferent between the two regimes.
Proof of Proposition 7.

First, we prove that the SV regime will defeat the PP regime. The SV regime and the PP regime are the same in that the high income individuals still have to "pay twice" if they want to use private education. In the proof of Proposition 6 we have established that the median voter under the SV regime will not buy any supplements to the voucher he receives. Hence his optimization problem is exactly the same as (A3). The value of g under the SV regime must satisfy the first-order condition: \( Y_a M(g, (pg/Y_a) j^Y_{P'(g)} \phi(Y) dY) = MC(g) \), where \( M(g,t) \) is defined in equation (2) and \( MC(g) \) in equation (3). Comparative statics on this condition reveals that

\[
\frac{\partial g}{\partial p} = \frac{Y_a (\partial M/\partial p) - (\partial MC/\partial p)}{(\partial MC/\partial g) - Y_a (\partial M/\partial g)}
\]

Recall that \( \partial MC/\partial g > 0 \) by assumption, and that \( \partial M/\partial g < 0 \), \( \partial M/\partial t < 0 \) and \( \partial M/\partial p < 0 \) because of diminishing marginal rate of substitution. From equation (3) it can be verified that \( \partial MC/\partial p > 0 \). Therefore, \( \partial g/\partial p < 0 \). In other words, the reduction in \( p \) under the SV regime will lead to an increase in \( g \), i.e., \( g_{SV} > g^* \).

The reduction in \( p \) will change the tax rate as well. Use the government budget constraint we can rewrite the first-order condition to (A3) as \( Y_a M(g, (1-t)Y) = MC(g) \). To maintain this equality, \( t \) has to fall as \( g \) goes up. Hence \( t_{SV} < t^* \).

Therefore, compared with the PP regime, the SV regime will bring a higher \( g \) and a lower tax rate. Individuals who use the public education prefer the SV regime to the PP regime. Individuals who use the private education also prefers the SV
regime to the PP regime because the SV regime entails a lower p and a lower t. Hence the SV regime will defeat the PP regime in a referendum.

Second, we prove that SV regime defeats the UV regime under the conditions specified in the Proposition. Recall from the proof of Proposition 6 that given p, individuals are indifferent between the SV regime and the PP regime. From the proof of Proposition 5 we know that given p, individuals whose $Y \leq Y_{uv}$ prefer the PP regime to UV regime. Therefore, given the same price p, these individuals also prefer the SV regime to the UV regime. Under either condition a) or condition b), those individuals constitute a majority. Thus, the SV regime will defeat the UV regime.
Figure 1

Figure 2
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